Rise of the Machines: Algorithmic Trading in the Foreign Exchange Market

ALAIN P. CHABOUD,   BENJAMIN CHIQUOINE,   ERIK HJALMARSSON, and CLARA VEGA*

ABSTRACT

We study the impact of algorithmic trading in the foreign exchange market using a long time series of high-frequency data that identify computer-generated trading activity. We find that algorithmic trading causes an improvement in two measures of price efficiency: the frequency of triangular arbitrage opportunities and the autocorrelation of high-frequency returns. We show that the reduction in arbitrage opportunities is associated primarily with computers taking liquidity. This result is consistent with the view that AT improves informational efficiency by speeding up price discovery, but that it may also impose higher adverse selection costs on slower traders. In contrast, the reduction in the autocorrelation of returns owes more to the algorithmic provision of liquidity. We also find evidence consistent with the strategies of algorithmic traders being highly correlated. This correlation, however, does not appear to cause a degradation in market quality, at least not on average.

JEL Classification: F3, G12, G14, G15. Keywords: algorithmic trading; price discovery; liquidity; high frequency trading; foreign exchange;

The use of algorithmic trading (AT), where computers monitor markets and manage the trading process at high frequency, has become common in major financial markets in recent years, beginning in the U.S. equity market in the 1990s. Since the introduction of AT, there has been widespread interest in understanding the potential impact it may have on market dynamics, particularly recently following several trading disturbances in the equity market blamed on computer-driven trading. While some have highlighted the potential for more efficient price discovery, others have expressed concern that it may lead to higher adverse selection costs and excess volatility.[1] In this paper, we analyze the effect that algorithmic ("computer") trades and non-algorithmic ("human") trades have on the informational efficiency of foreign exchange prices. This is the first formal empirical study on the subject in the foreign exchange market.

We rely on a novel data set consisting of several years (September 2003 to December 2007) of minute-by-minute trading data from Electronic Broking Services (EBS) in three currency pairs: euro-dollar, dollar-yen, and euro-yen. The data represent a large share of spot interdealer transactions across the globe in these exchange rates, with EBS widely considered to be the primary site of price discovery in these currency pairs during our sample period. A crucial feature of the data is that the volume and direction of human and computer trades are explicitly identified, allowing us to measure their respective impacts at high frequency. Another useful feature of the data is that they span the introduction and rapid growth of AT in an important market where it had not been previously allowed.

The theoretical literature highlights two main differences between computer and human traders. First, computers are faster than humans, both in processing information and in acting on that information. Second, there is the potential for higher correlation in computers' trading actions than in those of humans, as computers need to be pre-programmed and may react similarly to a given signal. There is no agreement, however, on the impact that these features of AT may have on the price discovery process.

Biais, Foucault, and Moinas (2011) and Martinez and Roşu (2011) argue that the speed advantage of algorithmic traders over humans – specifically, their ability to react more quickly to public information – should have a positive effect on the informativeness of prices. In their theoretical models, algorithmic traders are better informed than humans and use market orders to exploit their information. Given these assumptions, the authors show that the presence of algorithmic traders makes asset prices more informationally efficient, but, importantly, their trades are a source of adverse selection for those who provide liquidity. The authors argue that algorithmic traders contribute to price discovery because, once price inefficiencies arise, AT quickly makes them disappear by trading on posted quotes. Similarly, in the context of traders who implement "convergence trade" strategies (i.e., strategies taking long-short positions in assets with identical underlying cash flows that temporarily trade at different prices), Oehmke (2009) and Kondor (2009) argue that the higher the number of traders who implement those strategies, the more efficient prices will be. One could also argue, as does Hoffman (2013), that better-informed algorithmic traders who specialize in providing liquidity make prices more informationally efficient by posting quotes that reflect new

information quickly, thus preventing arbitrage opportunities from occurring in the first place.

In contrast to these mostly positive views on AT and price efficiency, Foucault, Hombert and Roşu (2013) argue that, in a world with no asymmetric information, the speed advantage of algorithmic traders would not increase the informativeness of prices but would still increase adverse selection costs. In their survey article, Biais and Woolley (2011) point out that the potential commonality of trading actions amongst computers may have a negative effect on the informativeness of prices. Khandani and Lo (2007, 2011), who analyze the large losses that occurred for many quantitative long-short equity strategies at the beginning of August 2007, highlight the possible adverse effects on the market of such commonality in behavior across market participants (algorithmic or not) and provide empirical support for this concern.[2] Kozhan and Tham (2012), in contrast to Oehmke's (2009) and Kondor's (2009) more standard notion that competition improves price efficiency, argue that computers entering the same trade at the same time to exploit an arbitrage opportunity could cause a crowding effect that pushes prices away from their fundamental values. Stein (2009) also highlights this potential crowding effect in the context of hedge funds simultaneously implementing "convergence trade" strategies.

Guided by this literature, our paper studies the impact of algorithmic trading on the price discovery process in the foreign exchange market. Besides giving us a measure of the relative participation of computers and humans in trades, our data also allows us to study more precisely how AT affects price discovery. In particular, we study separately the impact of trades where computers are providing liquidity to the market and trades where computers are taking liquidity from the market, addressing one of the questions posed in the literature. We also look at how the share of the market's order flow generated by algorithmic traders impacts price discovery. Finally, to address another concern highlighted in the literature— namely, that the trading strategies used by computers are more correlated than those used by humans, potentially creating excessive volatility— we propose a novel way of indirectly inferring the correlation among computer trading strategies from our trading data. The primary idea behind the measure we design is that traders who follow similar trading strategies trade less with each other than those who follow less correlated strategies.

We first use our data to study the impact of AT on the frequency of triangular arbitrage opportunities amongst the euro-dollar, dollar-yen, and euro-yen currency pairs. These arbitrage opportunities are a clear example of prices not being informationally efficient in the foreign exchange market. We document that the introduction and growth of algorithmic trading coincided with a substantial reduction in triangular arbitrage opportunities. We then continue with a formal analysis of whether AT activity causes a reduction in triangular arbitrage opportunities, or whether the relationship is merely coincidental, possibly due to a concurrent increase in trading volume or decrease in price volatility. To that end, we formulate a high-frequency vector autoregression (VAR) specification that models the interaction between triangular arbitrage opportunities and AT (the overall share of algorithmic trading and the various facets of algorithmic activity discussed above). In the VAR specification, we control for time trends, trading volume, and exchange rate return volatility in each currency pair. We estimate both a reduced-form of the VAR and a structural

VAR that uses the heteroskedasticity identification approach developed by Rigobon (2003), and Rigobon and Sack (2003, 2004).[3] In contrast to the reduced-form Granger causality tests, which essentially measure predictive relationships, the structural VAR estimation allows for identification of the contemporaneous causal impact of AT on triangular arbitrage opportunities.

Both the reduced-form and structural-form VAR estimations show that AT activity causes a reduction in the number of triangular arbitrage opportunities. In addition, we find that algorithmic traders reduce arbitrage opportunities more by acting on the quotes posted by non-algorithmic traders than by posting quotes that are then traded upon. This result is consistent with the view that AT improves informational efficiency by speeding up price discovery, but it increases the adverse selection costs to slower traders, as suggested by the theoretical models of Biais, Foucault, and Moinas (2011) and Martinez and Roşu (2011). We also find that a higher degree of correlation amongst AT strategies reduces the number of arbitrage opportunities over our sample period. Thus, contrary to Kozhan and Tham (2012), in this particular example commonality in trading strategies appears to be beneficial to the efficiency of the price discovery process.

The impact of AT on the frequency of triangular arbitrage opportunities is, however, only one facet of how computers may affect the price discovery process. More generally, we investigate whether algorithmic trading contributes to the temporary deviation of asset prices from their fundamental values, resulting in excess volatility, particularly at high frequencies. To investigate the effect of AT on this aspect of informational efficiency, we study the autocorrelation of high-frequency returns in our three exchange rates.[4] We again estimate both a reduced-form and a structural VAR and find that, on average, an increase in AT participation in the market causes a reduction in the degree of autocorrelation of high-frequency returns. Interestingly, we find that the improvement in the informational efficiency of prices now seems to come predominantly from an increase in the trading activity of algorithmic traders when they are providing liquidity— that is, posting quotes that are hit— not from an increase in the trading activity of algorithmic traders who hit posted quotes. In other words, in this case, in contrast to the study of triangular arbitrage, algorithmic traders appear to increase the informational efficiency of prices by posting quotes that reflect new information more quickly, consistent with Hoffman (2013).

Also in contrast to the triangular arbitrage case, we do not find a statistically significant association between a higher correlation of algorithmic traders' actions and high-frequency return autocorrelation. The difference between the autocorrelation results and those for triangular arbitrage may offer some support for one of the conclusions of Foucault (2011), namely that the effect of AT on the informativeness of prices may ultimately depend on the type of strategies used by algorithmic traders rather than on the presence of AT per se.

The paper proceeds as follows. In Section I, we briefly discuss the related empirical literature on the effect of algorithmic trading on market quality. Section II introduces the high-frequency data used in this study, including a short description of the structure of the market and an overview of the growth of AT in the foreign exchange market over time. In Section III, we present our measures of the various aspects of AT, including whether actions by computer traders appear to

be more correlated than those of human traders. In Sections IV and V, we test whether there is evidence that AT activity has a causal impact on the informativeness of prices, looking first at triangular arbitrage and then at high-frequency return autocorrelation. Finally, Section VI concludes. Additional clarifying and technical material is found in Appendices A to C.

I.   Related Empirical Literature

Our study contributes to the new empirical literature on the impact of AT on various measures of market quality, with the vast majority of the studies focusing on equity markets. A number of these studies use proxies to measure the share of AT in the market. Hendershott, Jones, and Menkveld (2011), in one of the earliest studies, use the flow of electronic messages on the NYSE after the implementation of Autoquote as a proxy for AT. They find that AT improves standard measures of liquidity (the quoted and effective bid-ask spreads). They attribute the decline in spreads to a decline in adverse selection — a decrease in the amount of price discovery associated with trading activity and an increase in the amount of price discovery that occurs without trading. The authors' interpretation of the empirical evidence is that computers enhance the informativeness of quotes by more quickly resetting their quotes after news arrivals. Boehmer, Fong, and Wu (2012) use an identification strategy that is similar to that of Hendershott, Jones, and Menkveld (2011). However, rather than using the implementation of Autoquote, they use the first availability of co-location facilities around the world to identify the effect that AT activity has on liquidity, short-term volatility, and the informational efficiency of stock prices. Hendershott, and Riordan (2012) use one month of AT data in the 30 DAX stocks traded on the Deutsche Boerse and find that algorithmic traders improve market liquidity by providing liquidity when it is scarce and consuming it when it is plentiful.

Some recent studies focus more specifically on the impact of high-frequency trading (HFT), generally viewed as a subset of algorithmic trading (not all algorithmic traders trade at extremely high frequency). For instance, Brogaard, Hendershott, and Riordan (2012) use NASDAQ data from 2008 and 2009 and find that high-frequency traders (HFTs) facilitate price efficiency by trading in the direction of permanent price changes during macroeconomic news announcement times and at other times. Using data similar to those of Brogaard, Hendershott, and Riordan (2012), Hirschey (2011) finds that aggressive purchases (sales) by HFTs predict future aggressive purchases (sales) by non-HFTs. Both of these studies suggest that HFTs are the "informed" traders. Hasbrouck and Saar (2012) develop an algorithm to proxy for overall HFT activity and conclude that HFT activity reduces short-term volatility and bid-ask spreads, and increases displayed depth in the limit order book. Menkveld (2011) analyzes the trading strategy of one large HFT in the Chi-X market and concludes that the HFT behaves like a fast version of the classic market maker. Kirilenko et al. (2011) study the role HFTs played during the May 6, 2010 "flash crash" and conclude that HFTs did not trigger the crash, but that they contributed to the crash by pulling out of the market as market conditions became challenging.

Our work complements these studies along several dimensions. First, we study a different asset class, foreign exchange, which is traded in a very large global market.[5] Second, we have a long data set that clearly identifies computer-generated trades without appealing to proxies, and that spans the introduction of AT in the market. Third, the data also permits us to differentiate the effects of certain features of AT, separating, for instance, trades initiated by a computer and trades initiated by a human, which in turn allows us to address recently-developed theories on how AT impacts price discovery. Fourth, we have data on three interrelated exchange rates, and thus can analyze the impact of computer trades on the frequency of a very obvious type of arbitrage opportunity. Finally, we study the correlation of AT strategies and its impact on the informational efficiency of prices, also relating our findings to the recent theoretical literature.

II. Data Description

A. Market Structure

Over our 2003 to 2007 sample period, two electronic platforms processed a majority of global interdealer spot trading in the major currency pairs, one offered by Reuters and one offered by Electronic Broking Services (EBS). Both of these trading platforms are electronic limit order books. Importantly, trading in each major currency pair is highly concentrated on only one of the two systems. Of the most traded currency pairs (exchange rates), the top two, euro-dollar and dollar-yen, trade primarily on EBS, while the third, sterling-dollar, trades primarily on Reuters. As a result, price discovery for the spot euro-dollar, for instance, occurs on the EBS system, and dealers across the globe base their spot and derivative quotes on that price. EBS controls the network and each of the terminals on which the trading is conducted. Traders can enter trading instructions manually, using an EBS keyboard, or, upon approval, via a computer directly interfacing with the system. The type of trader (human or computer) behind each trading instruction is recorded by EBS, allowing for our study.

The EBS system is an interdealer system accessible to foreign exchange dealing banks and, under the auspices of dealing banks (via prime brokerage arrangements), to hedge funds and commodity trading advisors (CTAs). As it is a "wholesale" trading system, the minimum trade size over our sample period is one million of the "base" currency, and trade sizes are only allowed in multiples of millions of the base currency. We analyze data in the three most traded currency pairs on EBS, euro-dollar, dollar-yen, and euro-yen.[6]

B. Quote and Transaction Data

Our data consist of both quote data and transactions data. The quote data, at the one-second frequency, consist of the highest bid quote and the lowest ask quote on the EBS system in our three currency pairs. The quote data are available from 1997 through 2007. All the quotes are executable and therefore truly represent the market price at that instant. From these data, we construct mid-quote series from which we compute exchange rate returns at various frequencies.

The transactions data, available from October 2003 through December 2007, are aggregated by EBS at the one-minute frequency. Throughout the subsequent analysis, we focus on data sampled between 3am and 11am New York time, which represent the most active trading hours of the day (see Berger et al. (2008) for further discussion on trading activity on the EBS system). That is, each day in our sample is made up of the intradaily observations between 3am and 11am New York time.[7]

The transaction data provide detailed information on the volume and direction of trades that can be attributed to computers and humans in each currency pair. Specifically, each minute we observe trading volume and order flow for each of the four possible pairs of human and computer makers and takers: human maker/human taker (HH), computer maker/human taker (CH), human maker/computer taker (HC), and computer maker/computer taker (CC).[8] Order flow is defined, as is common, as the net of buyer-initiated trading volume minus seller-initiated trading volume, with traders buying or selling the base currency. We denote the trading volume and order flow attributable to any maker taker pair as $Vol(\cdot)$ and $OF(\cdot)$, respectively.

Figure 1 shows for each currency pair over the 2003 to 2007 period, the percent of trading volume where at least one of the two counterparties is an algorithmic trader. We label this variable $VAT = 100 \times \frac{Vol(CH+HC+CC)}{Vol(HH+CH+HC+CC)}$. The fraction of trading volume involving AT for at least one of the counterparties grows from zero in 2003 to near 60% in 2007 for euro-dollar and dollar-yen trading, and to nearly 80% for euro-yen trading.

[Figure 1 here]

Figure 2 shows the evolution over time of the four types of trades, $Vol(HH)$, $Vol(CH)$, $Vol(HC)$, and $Vol(CC)$, as fractions of total volume. By the end of 2007, in the euro-dollar and dollar-yen markets, human to human trades (the solid lines) account for slightly less than half of the volume, and computer to computer trades (the dotted lines) for about 10% to 15%. In these two currency pairs, $Vol(CH)$ is often close to $Vol(HC)$, that is, computers take prices posted by humans (the dashed lines) about as often as humans take prices posted by market-making computers (the dotted-dashed lines). The story is different for the cross-rate, the euro-yen currency pair. By the end of 2007, there are more computer-to-computer trades than human-to-human trades. But the most common type of trade in euro-yen is computers trading on prices posted by humans. We believe this reflects computers taking advantage of short-lived triangular arbitrage opportunities, where prices set in the euro-dollar and dollar-yen markets, the primary sites of price discovery, are very briefly out of line with the euro-yen cross-rate. Detecting and trading on triangular arbitrage opportunities is widely thought to have been one of the first strategies implemented by algorithmic traders in the foreign exchange market, which is consistent with the more rapid growth in algorithmic activity in the euro-yen market documented in Figure 1. We discuss the evolution of the frequency of triangular arbitrage opportunities in Section IV of the paper.

[Figure 2 here]

III.   Measuring Various Features of Algorithmic Trading

A.   Inferring The Correlation of Algorithmic Strategies from Trade Data: The R-Measure

There is little solid information, and certainly no data, about the precise mix of strategies used by algorithmic traders in the foreign exchange market, as traders and EBS keep what they know confidential. From conversations with market participants, however, we believe that over our sample period, about half of the AT volume on EBS comes from traditional foreign exchange dealing banks, with the other half coming from other financial firms, including hedge funds and CTAs. Those other financial firms, which access EBS under prime-brokerage arrangements, can only trade algorithmically over our sample period. Some of the banks' computer trading in our sample is related to activity on their own customer-to-dealer platforms, to automated hedging activity, and to the optimal execution of large orders. But a sizable fraction (perhaps almost a half) is believed to be proprietary trading using a mix of strategies similar to what hedge funds and CTAs use. These strategies include various types of high-frequency arbitrage, including some across different asset markets, a number of lower-frequency statistical arbitrage strategies (including carry trades), and strategies designed to automatically react to news and data releases (still fairly rare in the foreign exchange market as of 2007). Overall, market participants believe that the main difference between the mix of algorithmic strategies used in the foreign exchange market and the mix used in the equity market is that optimal execution algorithms are less prevalent in foreign exchange than in equity.

As mentioned previously, a concern expressed by the literature about algorithmic traders is that their strategies, which have to be pre-programmed, may be less diverse and hence more correlated than those of humans.[9] Computers could then react in the same fashion, at the same time, to the same information, creating excess volatility in the market, particularly at high frequency. While we do not observe the trading strategies of our market participants, we can derive some information about the correlation of algorithmic strategies from the trading activity of computers and humans. The idea is as follows. Traders who follow similar trading strategies and therefore send similar trading instructions at the same time will trade less with each other than those who follow less correlated strategies. Therefore, the extent to which computers trade (or do not trade) with each other should contain information about how correlated their algorithmic strategies are.

To extract this information, we first consider a simple benchmark model that assumes random and independent matching of traders. This is a reasonable assumption given the lack of discrimination between manual traders and algorithmic traders in the EBS matching process, that is, EBS does not differentiate in any way between humans and computers when matching buy and sell orders in its electronic order book. Traders also do not know the identity and type of trader they have been matched with until after the full trade is complete. The model allows us to determine the theoretical probabilities of the four possible types of trades: human maker/human taker, computer maker/human taker, human maker/computer taker, and computer maker/computer taker. We then compare these theoretical probabilities to those observed in the actual trading data. The bench-

mark model is fully described in Appendix B. Below we outline the main concepts and empirical results.

Under our random and independent matching assumption, computers and humans, both of which are indifferent ex-ante between making and taking, trade with each other in proportion to their relative presence in the market. For instance, in a world with more human trading activity than computer trading activity (which is the case in the vast majority of our sample), we should observe that computers take more liquidity from humans than from other computers. That is, the probability of observing human maker/computer taker trades, $Prob(HC)$, should be larger than the probability of observing computer maker/computer taker trades, $Prob(CC)$. We label the ratio of the two, $Prob(HC)/Prob(CC)$, the computer taker ratio, RC. Similarly, under our assumption, in such a world one would expect humans to take more liquidity from other humans than from computers, that is, $Prob(HH)$ should be larger than $Prob(CH)$. We label this ratio, $Prob(HH)/Prob(CH)$, the human taker ratio, RH. In summary, in our data with more human trading activity than computer trading activity, one would expect to find that $RC > 1$ and $RH > 1$.

Importantly, however, irrespective of the relative prevalence of human and computer trading activity, the benchmark model predicts that the ratio of these two ratios, the computer taker ratio divided by the human taker ratio, should be equal to one. That is, under random matching, the model predicts $R = RC/RH = 1$, with computers taking liquidity from humans in the same *proportion* that humans take liquidity from other humans.

The R-measure described above implicitly takes into account the commonality of trading direction to the extent that the matching process of the electronic limit order book takes it into account. For instance, two computers sending instructions to buy the euro at the same time will obviously not be matched by the electronic order book. In contrast, if about half of the algorithmic traders are buying and about half are selling, algorithmic traders will have a high probability of being matched with each other. Still, the buying and selling behavior of computers is an additional variable our data allows us to condition on when we infer whether computers' trading actions are highly correlated.[10] Hence, in Appendix B, we also describe in detail a model that explicitly takes the trading direction of computers into account. Using notation similar to the model without trading direction, this model yields the ratio of the buy ratios, $R^B = RC^B/RH^B$, and the ratio of the sell ratios, $R^S = RC^S/RH^S$. Again, under random matching, the benchmark model predicts that these ratios are both equal to one.

Based on the model described above, we calculate R, R^S, and R^B for the three currency pairs in our 2003 to 2007 sample period at one-minute, five-minute, and daily frequencies. Specifically, the realized values of RH and RC are given by $\widehat{RH} = \frac{Vol(HH)}{Vol(CH)}$ and $\widehat{RC} = \frac{Vol(HC)}{Vol(CC)}$, where, for instance, $Vol\left(HC\right)$ is either the one-minute, five-minute or daily trading volume between human makers and computer takers, following the notation described in Section II. Similarly, we define $\widehat{RH^S} = \frac{Vol\left(HH^S\right)}{Vol(CH^S)}$, $\widehat{RH^B} = \frac{Vol\left(HH^B\right)}{Vol(CH^B)}$, $\widehat{RC^S} = \frac{Vol\left(HC^S\right)}{Vol(CC^S)}$, and $\widehat{RC^B} = \frac{Vol\left(HC^B\right)}{Vol(CC^B)}$, where $Vol\left(HH^B\right)$ is the buy volume between human makers and human takers (i.e., the trading volume that involves buying of the base currency by the taker), $Vol\left(HH^S\right)$ is the sell volume between human makers

and human takers, and so forth.

Table I shows the means of the natural log of the one-minute, five-minute, and daily ratios of ratios, $\ln \widehat{R} = \ln(\frac{\widehat{RC}}{\widehat{RH}})$, $\ln \widehat{R^S} = \ln(\frac{\widehat{RC^S}}{\widehat{RH^S}})$, and $\ln \widehat{R^B} = \ln(\frac{\widehat{RC^B}}{\widehat{RH^B}})$, calculated for each currency pair in our data.[11] In contrast to the benchmark predictions that $R \equiv 1$, $R^B \equiv 1$ and $R^S \equiv 1$, or equivalently, that $\ln R \equiv 0$, $\ln R^B \equiv 0$ and $\ln R^S \equiv 0$, we find that, for all three currency pairs, at all frequencies, $\ln \widehat{R}$, $\ln \widehat{R^B}$ and $\ln \widehat{R^S}$ are substantially and significantly greater than zero. The table also shows the number of periods in which the statistics are above zero. At the daily frequency, $\ln \widehat{R}$, $\ln \widehat{R^B}$, and $\ln \widehat{R^S}$ are above zero for more than 95% of the days for all currencies. At all frequencies, the readings are highest for the euro-yen currency pair.[12]

[Table I here]

Table I provides clear evidence that the pattern of trading among humans and computers is different from that predicted by a random matching model. In particular, our finding that R is greater than one indicates that computers do not trade with each other as much as a random matching model would predict. This finding is consistent with the trading strategies of computers being correlated, which is the interpretation that we propose. However, we note that a finding of $R > 1$ is not unambiguous proof that computer strategies are correlated. Indeed, as a consequence of market clearing, $R > 1$ must also imply that humans do not trade with each other as much as the random matching model predicts, which could also reasonably be viewed as consistent with correlated human strategies. In each case, there could be an endogenous response from one group (humans or computers) to the correlated strategies of the other group, such that trading within each group is diminished. The fundamental cause that forces R to deviate from one is therefore not identified.

Still, we believe that the interpretation of $R > 1$ as evidence of correlated AT strategies is the most plausible. From conversations with market participants, there is widespread anecdotal evidence that in the very first years of AT in this market, a fairly limited number of strategies were being implemented, with triangular arbitrage among the most prominent. As mentioned above, Table I shows that the estimates of R are always highest for the euro-yen currency pair. This is consistent with the view that a large fraction of computer traders in the cross-rate were often attempting to take advantage of the same short-lived triangular arbitrage opportunities, and thus were more likely to send the same trading instructions at the same time in this currency pair, trading little with each other.[13] Furthermore, human participation remains higher than computer participation in almost all of our sample and it seems less plausible that a larger number of market participants could use the same strategies and react at the very same time.[14]

We next describe the other variables that we create to measure certain features of AT activity that we use in our analysis.

B. Measuring Other Features of Algorithmic Trading Activity

With our trading data providing minute-by-minute information on the volume of trade, direction of trade, and whether the trade is attributed to algorithmic (computer) or nonalgorithmic (human)

traders, we can study not just the overall impact of the presence of AT in the market but also the mechanism through which AT affects the market. To this end, we create several variables at the minute-by-minute frequency to use in the analysis. First, we measure overall AT activity as the fraction of total volume where at least one of the two counterparties of a trade is a computer. As explained above, we label this variable $VAT = 100 \times \frac{Vol(HC)+Vol(CH)+Vol(CC)}{Vol(HH)+Vol(HC)+Vol(CH)+Vol(CC)}$ and show its evolution over time in Figure 1. This is a more precise measure of AT activity than proxies used in recent studies, and is equivalent to what has been used in studies that had access to actual data on the fraction of AT in the market. Second, we measure the fraction of the overall trading volume where a computer is the aggressor in a trade, trading on an existing quote, in other words, the relative taking activity of computers. We label this variable $VCt = 100 \times \frac{Vol(HC)+Vol(CC)}{Vol(HH)+Vol(HC)+Vol(CH)+Vol(CC)}$. Third, we measure the fraction of the overall trading volume where a computer provides the quote hit by another trader, in other words, the relative making activity of computers. We label this variable $VCm = 100 \times \frac{Vol(CH)+Vol(CC)}{Vol(HH)+Vol(HC)+Vol(CH)+Vol(CC)}$. Fourth, we measure the share of order flow in the market coming from computer traders, relative to the order flow of both computer and human traders. This variable, which by the definition of order flow takes into account the sign of the trades initiated by computers, can be viewed as a measure of the relative "directional intensity" of computer-initiated trades in the market. We label this variable $OFCt = 100 \times \frac{|OF(C-Take)|}{|OF(C-Take)|+|OF(H-Take)|}$, where $OF(C-Take) = OF(HC) + OF(CC)$ and $OF(H-Take) = OF(HH) + OF(CH)$.[15]

We use these variables in addition to our R-measure to study more precisely how AT affects the process of price discovery. For instance, in our analysis of triangular arbitrage reported below, a finding that VCt is more strongly (negatively) related to triangular arbitrage opportunities than VCm would lead us to conclude that the empirical evidence provides some support for the conclusions of Biais, Foucault, and Moinas (2011), that algorithmic traders contribute to market efficiency by aggressively trading on existing arbitrage opportunities, therefore taking liquidity. An opposite finding would be evidence that algorithmic traders who specialize in providing liquidity are the ones who make prices more informationally efficient, by posting quotes that reflect information more quickly in the first place.

IV. Triangular Arbitrage and Algorithmic Trading

In this section we analyze the effects of AT on the frequency of triangular arbitrage opportunities in the foreign exchange market. We begin by providing graphical evidence suggesting that the introduction and growth of AT coincides with a reduction in triangular arbitrage opportunities, and then proceed with a more formal causal analysis.

A. Preliminary Graphical Evidence

Our data contain second-by-second bid and ask quotes on three related exchange rates (euro-dollar, dollar-yen, and euro-yen), allowing us to estimate the frequency with which these exchange

rates are "out of alignment." More precisely, each second we evaluate whether a trader, starting with a dollar position, could profit from purchasing euros with dollars, purchasing yen with euros, and purchasing dollars with yen, all simultaneously at the relevant bid and ask prices. An arbitrage opportunity is recorded for any instance in which such a strategy (or a "round trip" in the other direction) yields a strictly positive profit. In Figure 3 (top panel) we show the daily frequency of such triangular arbitrage opportunities from 2003 through 2007 as well as the frequency of triangular arbitrage opportunities that yield a profit of one basis point or more (bottom panel).[16]

[Figure 3 here]

The frequency of arbitrage opportunities clearly drops over our sample. For arbitrage opportunities yielding a profit of one basis point or more (Figure 3 bottom panel), the decline is particularly noticeable around 2005, when the rate of growth in AT is highest. In fact, by 2005 we begin to see entire days without arbitrage opportunities of that magnitude. On average in 2003 and 2004, the frequency of such arbitrage opportunities is about 0.5%. By 2007, at the end of our sample, the frequency is 0.03%. As seen in the top panel of Figure 3, the decline in the frequency of arbitrage opportunities with a profit strictly greater than zero is, as we would expect, more gradual, and the frequency does not approach zero by the end of our sample. In 2003 and 2004, these opportunities occur with a frequency of about 3%; by 2007, their frequency is about 1.5%. This simple analysis highlights the potentially important impact of AT in this market, but obviously does not prove that AT caused the decline, as other factors could have contributed to, or even driven, the drop in arbitrage opportunities. However, the findings line up well with (i) the anecdotal (but widespread) evidence that one of the first strategies implemented by algorithmic traders in the foreign exchange market aimed to detect and profit from triangular arbitrage opportunities and (ii) the view that, as more arbitrageurs attempt to take advantage of these opportunities, their observed frequency declines (Oehmke (2009) and Kondor (2009)).

B. Formal Analysis

We now attempt to more formally identify the relationship between AT and the frequency of triangular arbitrage opportunities, controlling for changes in the total volume of trade and market volatility, as well as general time trends in the data. We use the second-by-second quote data to construct a minute-by-minute measure of the frequency of triangular arbitrage opportunities. Specifically, following the approach described in the preceding section, for each second we calculate the maximum profit achievable from a "round trip" triangular arbitrage trade in either direction, starting with a dollar position. A minute-by-minute measure of the frequency of triangular arbitrage opportunities is then calculated as the number of seconds within each minute with a strictly positive profit.

AT and triangular arbitrage opportunities are likely determined simultaneously, in the sense that both variables have a contemporaneous impact on each other. OLS regressions of contemporaneous triangular arbitrage opportunities on contemporaneous AT activity are therefore likely biased and misleading. To overcome these potential difficulties, we estimate a VAR system that allows for both

Granger causality tests as well as contemporaneous identification through heteroskedasticity in the data over the sample period (Rigobon (2013) and Rigobon and Sack (2003, 2004)). Let Arb_t be the minute-by-minute measure of triangular arbitrage opportunities and AT_t^{avg} be average AT activity across the three currency pairs; AT_t^{avg} is used to represent the five previously defined measures of AT activity. Define $Y_t = (Arb_t, AT_t^{avg})$. The structural VAR system is

$$AY_t = \Phi(L)Y_t + \Lambda X_{t-1:t-20} + \Psi G_t + \epsilon_t. \tag{1}$$

Here, A is a 2×2 matrix specifying the contemporaneous effects, normalized such that all diagonal elements are equal to one, $\Phi(L)$ is a lag-function that controls for the effects of the lagged endogenous variables, $X_{t-1:t-20}$ consists of lagged control variables not modelled in the VAR (specifically, $X_{t-1:t-20}$ includes the sum of the volume of trade in each currency pair over the past 20 minutes and the volatility in each currency pair over the past 20 minutes, calculated as the sum of absolute returns over these 20 minutes), and G_t represent a set of deterministic functions of time t, capturing individual trends and intradaily patterns in the variables in Y_t (in particular, $G_t = \left(I_{\{t \in \text{1st month of sample}\}}, ..., I_{\{t \in \text{last month of sample}\}}, I_{\{t \in \text{1st half-hour of day}\}}, ..., I_{\{t \in \text{last half-hour of day}\}} \right)$, which captures long-term secular trends in the data by year-month dummy variables as well as intradaily patterns accounted for by half-hour dummy variables). The structural shocks to the system are given by ϵ_t, which, in line with standard structural VAR assumptions, are assumed to be independent of each other and serially uncorrelated at all leads and lags. The number of lags included in the VAR is set to 20. Equation (1) thus provides a very general specification, allowing for full contemporaneous interactions between triangular arbitrage opportunities and AT activity.

Before describing the estimation of the structural system, we begin the analysis with the reduced-form system,

$$Y_t = A^{-1}\Phi(L)Y_t + A^{-1}\Lambda X_{t-1:t-20} + A^{-1}\Psi G_t + A^{-1}\epsilon_t. \tag{2}$$

The reduced-form is estimated equation-by-equation using ordinary least squares, and Granger causality tests are performed to assess the effect of AT on triangular arbitrage opportunities (and vice versa). In particular, we test whether the *sum* of the coefficients on the lags of the causing variable is equal to zero. Since the sum of the coefficients on the lags of the causing variable is proportional to the long-run impact of that variable, the test can be viewed as a long-run Granger causality test. Importantly, the sum of the coefficients also indicates the estimated direction of the (long-run) relationship, such that the test is associated with a clear direction in the causation.

Table II shows the results, where the first five rows in each subpanel provide the Granger causality results. The first three rows show the sum of the coefficients on the lags of the causing variable, along with the corresponding χ^2 statistic and p-value. The fourth and fifth rows in each subpanel provide the results of the standard Granger causality test, where all of the coefficients on the lags of the causing variable are jointly equal to zero.[17] The top panel shows tests of whether AT causes triangular arbitrage opportunities, whereas the bottom panel shows tests of whether

triangular arbitrage opportunities cause algorithmic trading.

[Table II here]

We first note that the relative presence of AT in the market (VAT) one minute has a statistically significant negative effect on triangular arbitrage the next minute: AT Granger causes a reduction in triangular arbitrage opportunities.[18] As seen in the lower half of the table, there is also strong evidence that an increase in triangular arbitrage opportunities Granger causes an increase in algorithmic trading activity. These relationships are seen even more clearly (judging by the size of the test statistics) when focusing on the taking activity of computers (VCt and $OFCt$), where there is very strong dual Granger causality between these measures of AT activity and triangular arbitrage. The making activity of computers (VCm), on the other hand, does not seem to be impacted by triangular arbitrage opportunities in a Granger-causal sense, and computer making actually Granger causes an *increase* in abitrage opportunities (this result is reversed, however, in the contemporaneous identification, so we do not lend it much weight). One interpretation of these results is thus that increased AT activity this period (representing trading, of course, but probably also a higher level of monitoring) makes it less likely that prices will deviate from being arbitrage-free at the beginning of next period. Conversely, if prices are further away from arbitrage-free this period, arbitrage opportunities are more likely to exist at the beginning of next period, and algorithmic traders are more likely to enter the market by taking liquidity on the relevant side to exploit those opportunities.

In summary, these findings suggest that algorithmic traders improve the informational efficiency of prices by taking advantage of arbitrage opportunities and making them disappear quickly, rather than by posting quotes that prevent these opportunities from occurring (as would be indicated by a strong negative effect of VCm on reducing triangular arbitrage opportunities). These results are also in line with the theoretical models of Biais, Foucault, and Moinas (2011), Martinez and Roşu (2011), Oehmke (2009), and Kondor (2009).

The Granger causality results point to a strong dual causal relationship between AT (taking) activity and triangular arbitrage, namely, an increase in AT causes a reduction in arbitrage opportunities and an increase in arbitrage opportunities causes an increase in AT activity. Although Granger causality tests are informative, they are based on the reduced-form of the VAR, and do not explicitly identify the *contemporaneous* causal economic relationships in the model. We therefore also estimate the structural version of the model, using a version of the heteroskedasticity identification approach developed by Rigobon (2003) and Rigobon and Sack (2003, 2004). The basic idea of this identification scheme is that heteroskedasticity in the error terms can be used to identify simultaneous equation systems. If there are two distinct variance regimes for the error terms, this is sufficient for identification of the simultaneous structural VAR system defined in equation (1). In particular, if the covariance matrices under the two regimes are not proportional to each other, this is a sufficient condition for identification (Rigobon (2003)).

We rely on the observation that the variance of the different measures of AT participation in the market that we use in our analysis changes over time.[19] To capitalize on this fact, we split our

sample into two equal-sized subsamples, simply the first and second halves of the sample period. Although the variances of the shocks are surely not constant within these two subsamples, this is not crucial for the identification mechanism to work, as long as there is a clear distinction in (average) variance over the two subsamples, as discussed in detail in Rigobon (2003). Rather, the crucial identifying assumption, as Stock (2010) points out, is that the structural parameters determining the contemporaneous impact between the variables (that is, A in equation (1) above) are constant across the two variance regimes.[20] This is not an innocuous assumption, although it seems reasonable in our context as we have no reason to suspect (and certainly no evidence) that the underlying structural impact of the variables in our model changed over our sample period. Appendix C describes the mechanics of the actual estimation, which is performed via GMM, and also provides a more extensive discussion of the method. Appendix C also lists estimates of the covariance matrices across the two different regimes (subsamples). The estimates show that there is strong heteroskedasticity between the first and second halves of the sample, and that the shift in the covariance matrices between the two regimes is not proportional, which is the key identifying condition mentioned above. Furthermore, we are reassured by the fact that in the vast majority of the statistically significant cases, the heteroskedasticity identification results are confirmed by the Granger causality analysis.

Estimates of the relevant contemporaneous structural parameters in equation (1) are shown in the sixth row of each subpanel in Table II, with Newey-West standard errors given in parentheses below. The eighth row in the table, shown in bold, provides a measure of the economic magnitude of the coefficients, obtained by multiplying the estimated contemporaneous effect for a given variable by its standard deviation.[21] The magnitude of these estimates can be compared directly across the different measures of AT.

Overall, the contemporaneous effects are generally in line with those found in the Granger causality tests (with the exception of the sign of the impact of VCm, which switches). An increase in the presence of AT activity has a contemporaneous negative effect on triangular arbitrage opportunities, and an increase in triangular arbitrage simultaneously causes an increase in AT activity. The making activity (VCm) of algorithmic traders is now also associated with a reduction in arbitrage opportunities but, judging by the economic magnitude of the coefficients, $OFCt$ and $\ln(R)$ have by far the biggest impact on arbitrage opportunities. In other words, high relative levels of computer order flow and high correlation of algorithmic strategies result in the largest reductions in this type of informational inefficiency. The results are not unexpected, as together, they likely indicate that the correlated taking activity of computers in a similar direction plays an important role in extinguishing triangular arbitrage opportunities. It is also interesting to note that the economic impact of $\ln(R)$ is even higher than that of $OFCt$. Since a high value of $\ln(R)$ indicates that computers trade predominantly with humans, this is consistent with the view advanced by Biais, Foucault, and Moinas (2011) and Martinez and Roşu (2011) — namely, that AT contributes positively to price discovery, but that it may also increase the adverse selection costs of slower traders.

In terms of the actual absolute level of economic significance, the estimated relationships are fairly large. For instance, consider the estimated contemporaneous impact of relative computer taking activity in the same direction ($OFCt$) on triangular arbitrage. The contemporaneous effect is roughly -0.03, and the scaled economic effect is roughly -0.5. AT activity (including $OFCt$) is measured in percent and triangular arbitrage is measured as the number of seconds in a minute with an arbitrage opportunity. Thus, a one percentage point increase in the relative directional intensity of computer trading ($OFCt$) would, on average, reduce the number of seconds with a triangular arbitrage in a given minute by 0.03. This sounds like a small effect, but the (VAR residual) standard deviation of $OFCt$ is around 17%, which suggests that a typical one standard deviation move in $OFCt$ results in a 0.5 second decrease in arbitrage opportunities in each minute; this is the estimated economic effect shown in the table. As seen in the top panel of Figure 3, in any given minute, there are typically only between one and two seconds with an arbitrage opportunity (that is, between 1.5% and 3% of the time), so a reduction of 0.5 seconds is quite sizeable. A one-standard deviation increase in "computer correlation" (that is, in $\ln(R)$) causes an even larger estimated effect, with a resulting average reduction in arbitrage opportunities of 1.5 seconds per minute. A typical increase in overall AT participation (VAT) leads to a 0.2 second reduction in triangular arbitrage opportunities each minute. Going the other direction, the causal impact of triangular arbitrage opportunities on AT activity is also fairly large. A typical increase in triangular arbitrage opportunities results in an increase in computer taking intensity ($OFCt$) of approximately 4% and an increase in overall computer participation (VAT) of 2%.

In summary, both the Granger causality tests and the heteroskedasticity identification approach point to the same conclusion: AT causes a reduction in the frequency of triangular arbitrage opportunities. The taking activity of algorithmic traders seems to be an important mechanism through which this is accomplished, that is, by computers hitting existing quotes in the system, with most of these quotes posted by human traders. Intervals of highly correlated computer trading activity also lead to a reduction in arbitrage opportunities, presumably as computers react simultaneously and in the same fashion to the same price data. Finally, the presence of triangular arbitrage opportunities causes an increase in AT activity, presumably as computers dedicated to detect this type of opportunity "lay in wait" until they detect a profit opportunity and then begin to place trading instructions.[22]

V. High-Frequency Return Autocorrelation and Algorithmic Trading

The impact of AT on the frequency of triangular arbitrage opportunities is only one facet of how computers affect the price discovery process. More generally, an often-cited concern is that AT may contribute to the temporary deviation of asset prices from their fundamental values, resulting in excess volatility, particularly at high frequencies. We thus investigate the effect algorithmic traders have on a more general measure of prices not being informationally efficient: the autocorrelation of

high-frequency currency returns.[23] Specifically, we study the impact of AT on the absolute value of the first-order autocorrelation of five-second returns.[24] In Figure 4 we plot the 50-day moving average of the absolute value of the autocorrelation of high-frequency (that is, five-second) returns estimated on a daily basis. Over time, the absolute value of autocorrelation in the least liquid currency pair, euro-yen, has declined drastically from an average value of 0.3 in 2003 to 0.03 in 2007, and the sharp decline in 2005 coincides with the growth of AT participation. Again, this is only suggestive evidence of the impact of AT. The decline in the absolute value of autocorrelation in dollar-yen is also notable, from 0.14 in 2003 to 0.04 in 2007, but there is no systematic change in the absolute value of autocorrelation in the most liquid currency pair traded, euro-dollar. The graphical evidence suggests that it is possible that algorithmic trading may have a greater effect on the informational efficiency of asset prices in less liquid markets.

[Figure 4 here]

In this section, we adopt the same identification approach that we used for triangular arbitrage opportunities, using a five-minute frequency VAR (that is, we estimate the autocorrelation of five-second returns each five-minute interval) that allows for both Granger causality tests as well as contemporaneous identification through the heteroskedasticity in the data over the sample period.[25] However, since autocorrelation is measured separately for each currency pair (unlike the overall frequency of triangular arbitrage opportunities), we fit a separate VAR for each currency pair. AT activity is measured in the same five ways as before. We study the overall impact of the share of AT activity in the market, VAT, and also that of the relative taking activity of computers, VCt, the relative making activity of computers, VCm, the relative presence of computer order flow, $OFCt$, and the degree of correlation in computer trading activity (and therefore strategies), $\ln(R)$.

Let $|AC_t^j|$ and AT_t^j be the five-minute measures of the absolute value of autocorrelation in five-second returns and AT activity, respectively, in currency pair j, $j = 1, 2, 3$; as before, AT_t^j is used to represent either of the five measures of AT activity. The autocorrelations are expressed in percent. Define $Y_t^j = \left(|AC_t^j|, AT_t^j \right)$. The structural form of the system is given by

$$A^j Y_t^j = \Phi^j(L) Y_t^j + \Lambda^j X_{t-1:t-20}^j + \Psi^j G_t + \epsilon_t^j, \tag{3}$$

where A^j is the 2×2 matrix specifying the contemporaneous effects, normalized such that all diagonal elements are equal to 1. For currency pair j, $X_{t-1:t-20}^j$ includes the sum of the volume of trade over the past 20 minutes and the volatility over the past 20 minutes, calculated as the sum of absolute returns over these 20 minutes. Similar to our previous specification, G_t controls for secular trends and intradaily patterns in the data by year-month dummy variables and half-hour dummy variables, respectively. The structural shocks ϵ_t^j are assumed to be independent of each other and serially uncorrelated at all leads and lags. The number of lags included in the VAR is set to 20. The corresponding reduced-form equation is

$$Y_t = \left(A^j\right)^{-1} \Phi^j(L) Y_t^j + \left(A^j\right)^{-1} \Lambda^j X_{t-1:t-20}^j + \left(A^j\right)^{-1} \Psi^j G_t + \left(A^j\right)^{-1} \epsilon_t^j \tag{4}$$

Table III shows the estimation results for both the reduced-form (equation (4)) and structural-form (equation (3)) VARs. The left-hand panels show the results for tests of whether AT has a causal impact on the absolute value of autocorrelations in five-second returns, and the right-hand panels show tests of whether the level of absolute autocorrelation has a causal impact on AT activity. The results are laid out in the same manner as for the triangular arbitrage case, with the only difference that each currency pair now corresponds to a separate VAR regression. In particular, the first five rows in each panel show the results from Granger causality tests based on the reduced-form of the VAR in equation (4). The sixth, seventh, and eighth rows in each panel show the results from the contemporaneous identification of the structural form. Identification is again provided by the heteroskedasticity approach of Rigobon (2003) and Rigobon and Sack (2003, 2004). The estimation is performed in the same way as for the triangular arbitrage system, described in Appendix C. Appendix C also shows and discusses the estimates of the covariance matrices for the two subsamples used in the estimation.

[Table III here]

The results in the right-hand panels do not reveal a clear pattern for the impact of higher absolute autocorrelation on AT activity. In particular, the contemporaneous coefficients do not show a strong pattern of statistical significance and there is disagreement in some cases between the Granger causality results and the contemporaneous results.

The results in the left-hand panels of Table III are far more consistent across currencies and between the two estimation methods. First, the estimates clearly show that an increase in the share of AT participation (VAT) causes a decrease in the absolute autocorrelation of returns. As it did for triangular arbitrage, AT also appears to improve this measure of market efficiency. The Granger causality results show a role for both computer taking activity (VCt) and computer making activity (VCm) in the reduction of high-frequency autocorrelation, although the making results appear statistically stronger based on the significance and size of the test statistics. The results from the structural estimation show more clearly that the contemporaneous effect is much stronger for computer making than for computer taking. This is seen both in the statistical significance of the estimates and in the economic magnitude of the relationships. The structural evidence clearly indicates that, in contrast to the triangular arbitrage opportunity case, the improvement in the informational efficiency of prices now predominantly comes from an increase in the trading activity of algorithmic traders who provide liquidity (i.e., those who post quotes that are hit), and not from an increase in the trading activity of algorithmic traders who hit quotes posted by others.

In terms of economic magnitude, the causal relationship between AT (making) activity and autocorrelation in returns seems somewhat less important than that between AT (taking) activity and triangular arbitrage, but still not insignificant. Recall that autocorrelation entered the VAR in a percentage format, and note that the scaled contemporaneous coefficients for VCm (in the left-hand VCm panel of Table III) are roughly between -0.3 and -0.4. This suggests that a typical (one standard deviation) change in AT making activity results in a drop of -0.3 to -0.4 percentage points in the absolute autocorrelation of returns. As seen in Figure 4, the average autocorrelations

towards the end of the sample period are in the neighborhood of 5% for all three currency pairs. A third of a percentage point decrease in the autocorrelation is thus not a huge effect, but it does not appear irrelevant either.

Finally, it is also worth noting that in contrast to the triangular arbitrage case, the degree of correlation in computer trading strategies ($ln(R)$) and the relative presence of computer order flow ($OFCt$), both measures of commonality in algorithmic activity, do not appear to have a clear causal effect on autocorrelation.[26] As a matter of fact, when looking at the effects of these two variables, not one of the Granger or contemporaneous coefficients in our three currency pairs is statistically significant. In particular, there is no evidence that, on average, the high correlation of algorithmic strategies causes market inefficiencies, as measured by high-frequency autocorrelation.

VI.   Conclusion

Our paper studies the impact of AT on the price discovery process in the global foreign exchange market using a long times series of high-frequency trading data in three major exchange rates. After describing the growth of AT in this market since its introduction in 2003, we study its effect on two measures of price efficiency, the frequency of triangular arbitrage opportunities among our three exchange rates, and the presence of excess volatility, which we measure as the autocorrelation of high-frequency returns. Using reduced-form and structural VAR specifications, we show that, on average, the presence of AT in the market causes both a reduction in the frequency of arbitrage opportunities and a decrease in high-frequency excess volatility. Guided by the recent theoretical literature on the subject, we also study the mechanism by which algorithmic traders affect price discovery. We find evidence that algorithmic traders predominantly reduce arbitrage opportunities by acting on the quotes posted by nonalgorithmic traders. This result is consistent with the view that AT improves informational efficiency by speeding up price discovery, but that it may also increase the adverse selection costs to slower traders, as suggested by the theoretical models of Biais, Foucault, and Moinas (2011) and Martinez and Roşu (2011). In contrast, we show that the impact of AT on excess volatility is more closely related to algorithmic trades that provide liquidity. In this case, consistent with Hoffman (2013), the improvement in the informational efficiency of prices may come from the fact that algorithmic quotes reflect new information more quickly. We also provide evidence that is consistent with the actions and strategies of algorithmic traders being less diverse, and more correlated, than those of nonalgorithmic traders. We find no evidence, however, that a higher level of correlation among strategies leads to excess volatility. As a final caution, we note that our study is about the average effect of AT on the price discovery process in the foreign exchange market. Therefore, it cannot be ruled out that AT may, on rare but important occasions, contribute to excess volatility.

Appendix A Definition of Order Flow and Volume

The transactions data are broken down into categories specifying the maker and taker of the trades (human or computer), and the direction of the trades (buy or sell the base currency), for a total of eight different combinations. That is, the first transaction category may specify, say, the minute-by-minute volume of trade that results from a computer taker buying the base currency by "hitting" a quote posted by a human maker. We would record this activity as the human-computer buy volume. The human-computer sell volume is defined analogously, as are the other six buy and sell volumes that arise from the remaining combinations of computers and humans acting as makers and takers.

From these eight types of buy and sell volumes, we can construct, for each minute, trading volume and order flow measures for each of the four possible pairs of human and computer makers and takers: human maker/human taker (HH), computer maker/human taker (CH), human maker/computer taker (HC), and computer maker/computer taker (CC). The *sum* of the buy and sell volumes for each pair gives the volume of trade attributable to that particular combination of maker and taker (denoted as $Vol(HH)$ or $Vol(HC)$, for example). The *sum* of the buy volume for each pair gives the volume of trade attributable to that particular combination of maker and taker, where the taker is buying (denoted as $Vol(HH^B)$ or $Vol(HC^B)$, for example). The *sum* of the sell volume for each pair gives the volume of trade attributable to that particular combination of maker and taker, where the taker is selling (denoted as $Vol(HH^S)$ or $Vol(HC^S)$, for example). The *difference* between the buy and sell volume for each pair gives the order flow attributable to that maker taker combination (denoted as $OF(HH)$ or $OF(HC)$, for example). The sum of the four volumes, $Vol(HH + CH + HC + CC)$, gives the total volume of trade in the market. The sum of the four order flows, $OF(HH) + OF(CH) + OF(HC) + OF(CC)$, gives the total (marketwide) order flow.[27]

Throughout the paper, we use the expressions "volume" and "order flow" to refer to both the marketwide volume and order flow and the volume and order flows from other possible decompositions, with the distinction clearly indicated. Importantly, the data allow us to consider volume and order flow broken down by the type of trader who initiated the trade (human taker $(HH + CH)$ and computer taker $(HC + CC)$), by the type of trader who provided liquidity (human maker $(HH + HC)$ and computer maker $(CH + CC)$), and by whether there was any computer participation $(HC + CH + CC)$.

Appendix B How Correlated Are Algorithmic Trades and Strategies?

In the benchmark model, there are H_m potential human makers (the number of humans standing ready to provide liquidity), H_t potential human takers, C_m potential computer makers, and C_t potential computer takers. For a given period of time, the probability of a computer providing

liquidity to a trader is equal to $Prob(computer - make) = \frac{C_m}{C_m + H_m}$, which we label for simplicity as α_m, and the probability of a computer taking liquidity from the market is $Prob(computer - take) = \frac{C_t}{C_t + H_t} = \alpha_t$. The remaining makers and takers are humans, in proportions $(1 - \alpha_m)$ and $(1 - \alpha_t)$, respectively. Assuming that these events are independent, the probabilities of the four possible trades, human maker/human taker, computer maker/human taker, human maker/computer taker, and computer maker/computer taker, are

$$
\begin{aligned}
Prob(HH) &= (1 - \alpha_m)(1 - \alpha_t) \\
Prob(HC) &= (1 - \alpha_m)\alpha_t \\
Prob(CH) &= \alpha_m(1 - \alpha_t) \\
Prob(CC) &= \alpha_m\alpha_t.
\end{aligned}
$$

These probabilities yield the identity

$$
Prob(HH) \times Prob(CC) \equiv Prob(HC) \times Prob(CH),
$$

which can be rewritten as
$$
\frac{Prob(HH)}{Prob(CH)} \equiv \frac{Prob(HC)}{Prob(CC)}.
$$

We label the first ratio, $RH \equiv \frac{Prob(HH)}{Prob(CH)}$, the "human taker" ratio and the second ratio, $RC \equiv \frac{Prob(HC)}{Prob(CC)}$, the "computer taker" ratio. In a world with more human traders (both makers and takers) than computer traders, each of these ratios will be greater than one, because $Prob(HH) > Prob(CH)$ and $Prob(HC) > Prob(CC)$, that is, computers take liquidity more from humans than from other computers, and humans take liquidity more from humans than from computers. However, under the baseline assumptions of our random-matching model, the identity shown above states that the ratio of ratios, $R \equiv \frac{RC}{RH}$, will be equal to one. In other words, humans will take liquidity from other humans in a similar proportion that computers take liquidity from humans. Observing $R \neq 1$ therefore implies deviations from random matching. In particular, if computers trade less with each other and more with humans than expected, RC becomes larger than under random matching. By market clearing, this also implies that humans must trade less with each other and more with computers than expected, such that RH becomes smaller than under random matching. Therefore, R must be greater than one in this case. The economic interpretation of $R > 1$ is discussed in detail in the main text.

To explicitly take into account the sign of trades, we amend the benchmark model as follows: we assume that the probability of the taker buying an asset is α_B and the probability of the taker selling is $1 - \alpha_B$. We can then write the probability of the following eight events (assuming each

event is independent):

$$
\begin{aligned}
Prob(HH^B) &= (1-\alpha_m)(1-\alpha_t)\alpha_B \\
Prob(HC^B) &= (1-\alpha_m)\alpha_t\alpha_B \\
Prob(CH^B) &= \alpha_m(1-\alpha_t)\alpha_B \\
Prob(CC^B) &= \alpha_m\alpha_t\alpha_B \\
Prob(HH^S) &= (1-\alpha_m)(1-\alpha_t)(1-\alpha_B) \\
Prob(HC^S) &= (1-\alpha_m)\alpha_t(1-\alpha_B) \\
Prob(CH^S) &= \alpha_m(1-\alpha_t)(1-\alpha_B) \\
Prob(CC^S) &= \alpha_m\alpha_t(1-\alpha_B)
\end{aligned}
$$

These probabilities yield the identities

$$
\begin{aligned}
Prob(HH^B) \times Prob(CC^B) &\equiv Prob(HC^B) \times Prob(CH^B) \\
(1-\alpha_m)(1-\alpha_t)\alpha_B\alpha_m\alpha_t\alpha_B &\equiv (1-\alpha_m)\alpha_t\alpha_B\alpha_m(1-\alpha_t)\alpha_B \\
&\text{and} \\
Prob(HH^S) \times Prob(CC^S) &\equiv Prob(HC^S) \times Prob(CH^S) \\
(1-\alpha_m)(1-\alpha_t)(1-\alpha_B)\alpha_m\alpha_t(1-\alpha_B) &\equiv (1-\alpha_m)\alpha_t(1-\alpha_B)\alpha_m(1-\alpha_t)(1-\alpha_B),
\end{aligned}
$$

which can be rewritten as

$$
\begin{aligned}
\frac{Prob(HH^B)}{Prob(CH^B)} &\equiv \frac{Prob(HC^B)}{Prob(CC^B)} \\
&\text{and} \\
\frac{Prob(HH^S)}{Prob(CH^S)} &\equiv \frac{Prob(HC^S)}{Prob(CC^S)}
\end{aligned}
$$

We label the $RH^B \equiv \frac{Prob(HH^B)}{Prob(CH^B)}$ the "human taker buyer" ratio, $RC^B \equiv \frac{Prob(HC^B)}{Prob(CC^B)}$ the "computer taker buyer" ratio, $RH^S \equiv \frac{Prob(HH^S)}{Prob(CH^S)}$ the "human taker seller" ratio, and $RC^S \equiv \frac{Prob(HC^S)}{Prob(CC^S)}$ the "computer taker seller" ratio.

In a world with more human traders (both makers and takers) than computer traders, each of these ratios will be greater than one, because $Prob(HH^B) > Prob(CH^B)$, $Prob(HH^S) > Prob(CH^S)$, $Prob(HC^B) > Prob(CC^B)$, and $Prob(HC^S) > Prob(CC^S)$. That is, computers take liquidity more from humans than from other computers, and humans take liquidity more from humans than from computers. However, under the baseline assumptions of our random-matching model, the identity shown above states that the ratio of ratios, $R^B \equiv \frac{RC^B}{RH^B}$, will be equal to one, and that $R^S \equiv \frac{RC^S}{RH^S}$ will also be equal to one. Observed values for R^B or R^S that deviate from one again provide evidence against trading patterns following random matching, with the empirically

relevant case of $R^S, R^B > 1$ discussed in the main text.

Appendix C Heteroskedasticity Identification

A. Methodology

Restate the structural equation,

$$AY_t = \Phi\left(L\right)Y_t + \Lambda X_{t-1:t-20} + \Psi G_t + \epsilon_t,$$

and the reduced-form,

$$Y_t = A^{-1}\Phi\left(L\right)Y_t + A^{-1}\Lambda X_{t-1:t-20} + A^{-1}\Psi G_t + A^{-1}\epsilon_t.$$

Let Ω_s be the variance-covariance matrix of the reduced-form errors in variance regime s, $s = 1, 2$, which can be directly estimated by $\hat{\Omega}_s = \frac{1}{T_s}\sum_{t\in s}u_t u_t'$, where u_t are the reduced-form residuals. Let $\Omega_{\epsilon,s}$ be the diagonal variance-covariance matrix of the structural errors and the following moment conditions hold:

$$A\Omega_s A' = \Omega_{\epsilon,s}.$$

The parameters in A and $\Omega_{\epsilon,s}$ can then be estimated with GMM, using estimates of Ω_s from the reduced-form equation. Identification is achieved as long as the covariance matrices constitute a system of equations that is linearly independent. In the case of two regimes, the system is exactly identified. Standard errors are calculated using the Newey-West estimator.

It is useful to also provide additional intuitive discussion on the heteroskedasticity identification approach. As mentioned, the basic idea is that if the variance of the shocks in the system changes over time, but the structural parameters linking the variables to each other (that is, the parameters in A in the current notation) remain constant, the system may be identified. An intuitive way of thinking of the method is that the shift in variances provides that extra source of variation needed for identification in the presence of endogeneity. In that sense, the method is similar in concept to typical instrumental variables estimation and, as discussed in Rigobon and Sack (2004), the heteroskedasticity identification approach can indeed be interpreted as an instrumental variable method. Purely heuristically, one might think of the (nonproportional) shift in variances as a form of relevance condition, and the condition that the structural relationship captured by A is constant across variance regimes as a form of validity condition.[28]

As with the usual validity condition for instrumental variables, the stability of A is generally not testable. Thus, provided that there is sufficient change in the variances of the shocks, which is relatively straightforward to investigate, the key condition for the method to work is parameter stability in A. This is not a trivial condition to satisfy. In cases where there is a perceived "structural change," it might be difficult to argue that the variance of the structural shocks changed but the structural relationships remained constant. In other cases, where the shift in variances does not

seem immediately associated with a "structural change," the stability of the structural relationships seems more easily defensible. Ideally, one can point to a specific exogenous event that is likely to have affected the variances of the shocks but not the structural parameters in A; such events are, of course, rare. Our case, it would seem, is somewhere in the middle of these scenarios. There is not one specific exogenous event that is likely to have caused the observed heteroskedasticity in our data, but there is also no obvious structural change that is likely to have triggered a shift in structural relationships. A number of events on EBS during our sample period may have affected the variance of our AT measures, including technological changes that allowed for faster electronic monitoring and responses, and the fact that more and more nonbank participants gained access to the system via prime-brokerage. But these changes were gradual and we cannot point to exact dates when they began to have an important influence. Importantly, we do not believe that these changes have influenced the fundamental way in which trades impact price discovery, such that the assumption of a stable structural relationship seems reasonable.

As always, with issues of identification, it is nearly impossible to provide completely watertight arguments in any given situation. A more agnostic interpretation is to view the results as conditional on the structural relationship remaining constant. That is, provided the relationship is constant over time, the method delivers consistent estimates of the causal effects.

B. Covariance Matrix Estimates Across Subsamples

The heteroskedasticity identification approach requires that there exist two linearly independent variance regimes. This implies that the covariance matrices for the errors in the two regimes are not proportional to each other. Table CI shows the estimates of the covariance matrix for the reduced-form VAR residuals for $Y_t = (Arb_t, AT_t^{avg})$, across the first and second halves of the sample. As is clear, there is a strong increase in the variance of AT, measured as VAT, VCt, VCm, or $OFCt$. In contrast, the variance in triangular arbitrage opportunities is almost constant across the two subsamples and it is the change in the covariance matrix between the two subsamples is not proportional. The variance of $\ln(R)$, which is the fifth measure of AT activity that we use, is somewhat higher in the first subsample relative to the second subsample, although the distinction is less sharp than for the other measures of algorithmic trading. Since $\ln(R)$ is only observed when there are nonzero trades of all types, there are many missing observations (see footnote 12). During the minutes when $\ln(R)$ is not missing, there is a fairly sharp decrease in the variance of triangular arbitrage opportunities. The shift in the covariance matrix from the first subsample to the second is therefore not proportional for $(Arb_t, \ln(R)_t^{avg})$ either, although the identification does not appear as strong as for the other AT measures.

[Table CI here]

Table CII shows the corresponding results for the VAR residuals from the specification with AT and high-frequency autocorrelation, $Y_t^j = \left(|AC_t^j|, AT_t^j\right)$. This specification is estimated separately for each currency pair, and the table reports results by currency pairs and by measures of AT. The story is similar to that in Table CI, with strong evidence that the covariance matrix

does *not* shift in a proportional manner from the first to the second subsample. For the euro-dollar, the variance of the high-frequency autocorrelation increases somewhat from the first to the second subsample, but nowhere near as sharply as for the VAT, VCt, VCm, and $OFCt$ measures of AT and the shift in the covariance matrix is not close to proportional. For the euro-yen and dollar-yen, the variance in high-frequency autocorrelation actually decreases from the first to the second subsample, whereas the variance of VAT, VCt, VCm, and $OFCt$ all increase, so again there seems to be strong identification. The variance of $\ln(R)$ decreases for all three currency pairs, and does so proportionally with the variance in high-frequency autocorrelation for the euro-yen. Therefore, there seems to be weak or no identification for the euro-yen specification using $\ln(R)$. For the euro-dollar and dollar-yen, however, there is still identification for the $\ln(R)$ specifications.

[Table CII here]

Table CI

Covariances of the Residuals of the Bivariate VAR with Triangular Arbitrage and AT Activity

The table shows the covariance matrices for the residuals from the reduced-form VAR in equation (2), which is estimated separately for each of the following measures of AT activity averaged across currency pairs: overall AT participation (VAT), AT making participation (VCt), AT taking participation (VCt), AT taking participation (VCt), AT making participation (VCm), relative computer taking activity ($OFCt$), and the natural logarithm of AT trade correlation ($\ln(R)$). The VAR is estimated using one-minute data for the full sample from 2003 to 2007, spanning 1,067 days. The residuals are split up into two sub samples, covering the first 534 days and last 533 days, respectively, and the covariance matrices are calculated separately for these two subsamples. The final column in the table shows the ratios between the estimates from the second and the first subsamples.

	First half of sample	Second half of sample	Ratio (second/first)
		VAT	
$Var(Arb)$	11.52	11.07	0.96
$Var(VAT)$	119.26	237.65	1.99
$Cov(Arb, VAT)$	5.32	3.31	0.62
Number of observations	256246	255770	
		VCt	
$Var(Arb)$	11.52	11.07	0.96
$Var(VCt)$	76.14	223.00	2.93
$Cov(Arb, VCt)$	5.06	3.86	0.76
Number of observations	256246	255770	
		VCm	
$Var(Arb)$	11.52	11.07	0.96
$Var(VCm)$	51.79	151.30	2.92
$Cov(Arb, VCm)$	0.42	−0.01	−0.02
Number of observations	256246	255770	
		OFCt	
$Var(Arb)$	11.50	11.05	0.96
$Var(OFCt)$	196.80	360.28	1.83
$Cov(Arb, OFCt)$	9.68	4.18	0.43
Number of observations	255972	255716	
		$\ln(R)$	
$Var(Arb)$	16.05	10.61	0.66
$Var(\ln(R))$	1.35	1.05	0.78
$Cov(Arb, \ln(R))$	0.78	0.18	0.23
Number of observations	13670	163043	

Table CII

Covariances of the Residuals of the Bivariate VAR with High-Frequency Autocorrelation and AT Activity

The table shows the covariance matrices for the residuals from the reduced form VAR in equation (4), which is estimated separately for each currency pair and each of the following measures of AT activity: overall AT participation (VAT), AT taking participation (VCt), AT making participation (VCm), relative computer taking activity ($OFCt$), and the natural logarithm of AT trade correlation ($\ln(R)$). The VAR is estimated using five-minute data for the full sample from 2003 to 2007, spanning 1,067 days. The residuals are split up into two sub samples, covering the first 534 days and last 533 days, respectively, and the covariance matrices are calculated separately for these two subsamples. The final set of columns in the table shows the ratios between the estimates from the second and the first subsamples.

	First half of sample			Second half of sample			Ratio (second/first)		
	USD/EUR	JPY/USD	JPY/EUR	USD/EUR	JPY/USD	JPY/EUR	USD/EUR	JPY/USD	JPY/EUR
VAT									
$Var(ac)$	111.98	136.27	180.68	127.15	118.82	125.22	1.14	0.87	0.69
$Var(VAT)$	12.34	50.39	210.03	91.66	160.78	258.44	7.43	3.19	1.23
$Cov(ac,VAT)$	−0.22	−1.06	2.01	−3.95	−3.70	−2.31	18.02	3.49	−1.15
Number of observations	51264	51260	51019	51168	51167	51094			
VCt									
$Var(ac)$	111.97	136.30	180.70	127.20	118.85	125.22	1.14	0.87	0.69
$Var(VCt)$	4.84	23.11	143.71	66.35	131.13	277.61	13.72	5.67	1.93
$Cov(ac,VCt)$	0.03	−0.60	−1.70	−1.57	−1.16	−1.63	−57.73	1.94	0.96
Number of observations	51264	51260	51019	51168	51167	51094			
VCm									
$Var(ac)$	111.97	136.28	180.67	127.16	118.83	125.21	1.14	0.87	0.69
$Var(VCm)$	6.32	25.92	96.48	52.81	101.01	203.51	8.35	3.90	2.11
$Cov(ac,VCm)$	−0.27	−0.65	3.32	−3.54	−3.63	−1.00	12.98	5.60	−0.30
Number of observations	51264	51260	51019	51168	51167	51094			
$OFCt$									
$Var(ac)$	111.89	136.10	180.31	127.22	118.90	125.09	1.14	0.87	0.69
$Var(OFCt)$	252.23	386.55	587.54	620.48	664.80	791.08	2.46	1.72	1.35
$Cov(ac,OFCt)$	0.02	2.64	2.37	1.47	0.24	1.17	67.63	0.09	0.49
Number of observations	51095	50871	49935	51155	51121	50880			
$\ln(R)$									
$Var(ac)$	99.47	111.24	140.61	124.50	114.62	117.84	1.25	1.03	0.84
$Var(\ln(R))$	0.84	1.05	1.24	0.54	0.76	1.05	0.65	0.73	0.85
$Cov(ac,\ln(R))$	−0.07	−0.09	−0.09	−0.08	−0.09	−0.07	1.16	0.97	0.78
Number of observations	6114	5968	6732	44723	42298	37493			

REFERENCES

Berger, David W., Alain P. Chaboud, Sergey V. Chernenko, Edward Howorka, and Jonathan H. Wright, 2008, Order flow and exchange rate dynamics in electronic brokerage system data, *Journal of International Economics* 75, 93-109.

Biais, Bruno, Thierry Foucault, and Sophie Moinas, 2011, Equilibrium algorithmic trading, Working paper, Toulouse School of Economics.

Biais, Bruno, and Paul Woolley, 2011, High frequency trading, Working paper, Toulouse School of Economics, IDEI.

Boehmer, Ekkehart, Kingsley Y. L. Fong, and Juan Wu, 2012, Algorithmic trading and changes in firms' equity capital, Working paper, EDHEC.

Brogaard, Jonathan A., Terrence Hendershott, and Ryan Riordan, 2012, Algorithmic trading and price discovery, Working paper, U.C. Berkeley.

DeLong, Bradford, Andrei Shleifer, Lawrence H. Summers, and Robert J.Waldmann, 1990, Positive feedback investment strategies and destabilizing rational speculation, *Journal of Finance* 45, 379-395.

Ehrmann, Michael, Marcel Fratzscher, and Roberto Rigobon, 2011, Stocks, bonds, money markets and exchange rates: Measuring international financial transmission, *Journal of Applied Econometrics* 26, 948-974.

Fama, Eugene, 1965, The behavior of stock market prices, *Journal of Business* 38, 34-105.

Fama, Eugene, 1970, Efficient capital markets: A review of theory and empirical work, *Journal of Finance* 25, 383-417.

Foucault, Thierry, 2013, Algorithmic trading: Issues and preliminary evidence, in *Market Microstructure: Confronting Many Viewpoints* (John Wiley & Sons).

Foucault, Thierry, Johan Hombert, and Ioanid Roşu, 2013, News trading and speed, Working paper, HEC.

Froot, Kenneth A., David S. Scharfstein, and Jeremy C. Stein, 1992, Herd on the street: Informational inefficiencies in a market with short-term speculation, *Journal of Finance* 47, 1461-1484.

Hasbrouck, Joel J., and Gideon G. Saar, 2012, Low-latency trading, Working paper, NYU.

Hendershott, Terrence, Jones M. Charles, and Albert J. Menkveld, 2011, Does algorithmic trading improve liquidity? *Journal of Finance* 66, 1-33.

Hendershott, Terrence, and Ryan Riordan, 2012, Algorithmic trading and the market for liquidity, *Journal of Financial and Quantitative Analysis*, forthcoming.

Hirschey, Nicholas H., 2013, Do high frequency traders anticipate buying and selling pressure? Working paper, LBS.

Hoffman, Peter, 2013, A dynamic limit order market with fast and slow traders, Working paper, ECB.

Ito, Takatoshi, Kenta Yamada, Misako Takayasu, and Hideki Takayasu, 2012, Free lunch! Arbitrage opportunities in the foreign exchange markets, Working paper no 18541, NBER.

Khandani, Amir E., and Andrew W. Lo, 2007, What happened to the quants in August 2007? *Journal of Investment Management* 5, 5-54.

Khandani, Amir E., and Andrew W. Lo, 2011, What happened to the quants in August 2007? Evidence from factors and transactions data, *Journal of Financial Markets* 14, 1-46.

Kirilenko, Andrei A., Albert S. Kyle, Mehrdad Samadi, and Tugkan Tuzun, 2011, The flash crash: The impact of high frequency trading on an electronic market, Working paper, Federal Reserve Board of Governors.

Kondor, Peter, 2009, Risk in dynamic arbitrage: Price effects of convergence trading, *Journal of Finance* 64, 631-655.

Kozhan, Roman, and Wing Wah Tham, 2012, Execution risk in high-frequency arbitrage, *Management Science*, forthcoming.

Lanne, Markku, and Helmut Lutkepohl, 2008, Identifying monetary policy shocks via changes in volatility, *Journal of Money, Credit and Banking* 40, 1131-1149.

Martinez, Victor Hugo, and Ioanid Roşu, 2011, High frequency traders, news and volatility, Working paper, HEC.

Menkveld, Albert, 2011, High frequency trading and the new market makers, Working paper, VU University Amsterdam.

Oehmke, Martin, 2009, Gradual arbitrage, Working paper, Columbia University.

Primiceri, Giorgio E., 2005, Time varying structural vector autoregresions and monetary policy, *Review of Economic Studies* 72, 821-852.

Rigobon, Roberto, 2003, Identification through heteroskedasticity, *Review of Economics and Statistics* 85, 777-792.

Rigobon, Roberto, and Brian Sack, 2003, Measuring the reaction of monetary policy to the stock market, *Quarterly Journal of Economics* 118, 639-669.

Rigobon, Roberto, and Brian Sack, 2004, The impact of monetary policy on asset prices, *Journal of Monetary Economics* 51, 1553-1557.

Samuelson, Paul A., 1965, Proof that properly anticipated prices fluctuate randomly, *Industrial Management Review* 6, 41-49.

Stein, Jeremy C., 2009, Presidential address: Sophisticated investors and market efficiency, *Journal of Finance* 64, 1571-1548.

Stock, James H., 2010, The other transformation in econometric practice: Robust tools for inference, *Journal of Economic Perspective* 24, 83-94.

Vives, Xavier, 1994, Short-term investment and the informational efficiency of the market, *Review of Financial Studies* 8, 125-160.

Wright, Jonathan, 2012, What does monetary policy do to long-term interest rates at the zero lower bound?, *Economic Journal* 122, F447-F466.

Notes

[1]Biais and Wooley (2011) and Foucault (2011) provide excellent surveys of the recent literature on this topic.

[2]One could also argue that if algorithmic traders behave, as a group, like the positive-feedback traders of DeLong, Shleifer, and Summers (1990), or the chartists described in Froot and Stein (1992), or the short-term investors in Vives (1995), then their activity, in total, would likely create excess volatility.

[3]To identify the parameters of the structural VAR, we use the heteroskedasticity in AT activity over the sample.

[4]We use the term "excess volatility" to refer to any variation in prices that is not solely reflecting changes in the underlying efficient price process. Under the usual assumption that efficient prices follow a random walk specification (see discussion in footnote 23), autocorrelation in returns therefore imply some form of excess volatility and temporary deviations from the efficient price. When discussing the empirical results, we use the terms autocorrelation and excess volatility interchangeably.

[5]The structure of the wholesale spot foreign exchange market is obviously different from that of the major equity markets, but some of the differences probably facilitate the analysis of the impact of AT. In particular, while there are several platforms where one can trade foreign exchange in large quantities, price discovery in the currency pairs we study occurs overwhelmingly on one platform, EBS, the source of our data. Also, unlike a number of equity trading venues, traders do not get rebates for providing liquidity on EBS, which may allow for a "cleaner" study of the role of AT making and taking in price discovery. We note that, as this is a wholesale market, AT potentially causes a smaller increase in adverse selection costs than it may cause in the equity market. The minimum trade size is one million of the base currency, and thus there are no "small" market participants.

[6]The euro-dollar currency pair is quoted as an exchange rate in dollars per euro, with the euro the "base" currency. Similarly, the euro is also the base currency for euro-yen, while the dollar is the base currency for the dollar-yen pair.

[7]We exclude data collected from Friday 17:00 through Sunday 17:00 New York time from our

sample, as activity on the system during these "non-standard" hours is minimal and not encouraged by the foreign exchange community. Trading is continuous outside of the weekend, but the value date for trades, by convention, changes at 17:00 New York time, which therefore marks the end of each trading day. We also drop certain holidays and days of unusually light volume: December 24-December 26, December 31-January 2, Good Friday, Easter Monday, Memorial Day, Labor Day, Thanksgiving and the following day, and July 4 (or, if this is on a weekend, the day on which the U.S. Independence Day holiday is observed).

[8]The naming convention for "maker" and "taker" reflects the fact that the "maker" posts quotes before the "taker" chooses to trade at that price. Posting quotes is, of course, the traditional role of the market-"maker." The taker is also often viewed as the "aggressor" or the "initiator" of the trade. We refer the reader to Appendix A1 for more details on how we calculate volume and order flow for these four possible pairs of human and computer makers and takers.

[9]As pointed out by the referee, it is true that algorithms are necessarily rule-based at very high execution frequencies, but at a somewhat lower frequency, human monitoring is still common for a variety of algorithms.

[10]We thank an anonymous referee for this suggestion.

[11]We report summary statistics for the log-transformed variables because these results are invariant to whether one defines the R-measures as above, with a value greater than one consistent with correlated computer trading, or in the opposite direction (having Vol(CC) in the numerator), with a value less than one consistent with correlated computer trading. That is, the mean estimate will not be symmetric around one when inverting the R-measure, whereas the mean estimate of $\ln(R)$ will be symmetric around zero when inverting the R-measure.

[12]Note that the fraction of missing observation increases in all currency pairs as the period of observation over which the R-measure is calculated gets smaller. Missing observations arise mostly when $Vol(CC) = 0$, that is when computers do not trade with each other at all in a given time interval, which is more likely to occur in a shorter time interval. This can happen because computer trading activity is very small, as it is early in the sample, reducing the probability of a CC match. But it can also happen when there is more abundant computer trading activity but their strategies are so highly correlated that no CC match occurs. Excluding the information contained in this second type of missing observations means that, if anything, we may be underestimating the extent

of the correlation of algorithmic strategies in the data.

[13]Euro-yen is the currency pair with the smallest trading volume in our sample. A given triangular arbitrage trade, which automatically involves a similarly sized trade in our three exchange rates, would therefore represent a larger fraction of the total trades in euro-yen.

[14]Human participation remains higher than computer participation throughout our sample for euro-dollar and dollar-yen. In euro-yen, the cross rate, only the last few months of the 2003-2007 sample have a higher computer participation.

[15]One could also consider an analogously defined $OFCm$ variable as a measure of how correlated the actions of liquidity-making computers are. However, we chose not to add this variable to our analysis because the interpretation of the results seems ambiguous. For instance, if one found that an increase in $OFCm$ led to an increase in high-frequency return autocorrelation, we would still be hesitant to attribute this finding to the correlated actions of liquidity-making computers. Instead, because of the empirical evidence which shows that order flow from the liquidity taker's point of view drives asset prices, it would be more likely that the finding is driven by traders taking liquidity from computers in a correlated way.

[16]We take into account actual bid and ask prices in our triangular arbitrage calculations every second, which implies that the largest component of the transactions costs is accounted for in the calculations. But algorithmic traders likely incur other, more modest, costs, some fixed, some variable, but with the additional variable costs certainly well below 1 basis point in this market. We should not, however, expect all arbitrage opportunities with a positive profit to completely disappear. In the formal analysis we use a straightforward zero minimum profit level to calculate the frequency of arbitrage opportunities.

[17]Since the standard Granger causality test is not associated with a clear direction in causation, we focus our discussion on the long-run test reported in the first through third rows of Table II, which explicitly shows the direction of causation. Overall, the traditional Granger causality tests yield very similar results to the long-run tests in terms of statistical significance.

[18]The first lag of the causing variable tends to be the primary driver of the Granger causality relationship. Viewing the results as Granger causation from one minute to the next is therefore reasonable.

[19]The variance of VAT, VCt, VCm, and $OFCt$ increase over time as AT participation increases,

while the variance of the trade correlation measure, R, decreases over the sample period. Appendix C describes these results in detail.

[20]Stock (2010) provides an overview of the heteroskedasticity identification method, among other methods, in his discussion of new "robust tools for inference," classifying the heteroskedasticty identification method as a "quasi-experiment" approach. Some examples of recent studies that have used this method are Wright (2012), Ehrmann, Fratzscher, and Rigobon (2001), Lanne and Lutkepohl (2008), and Primiceri (2005).

[21]The coefficient for a given variable is scaled by the standard deviation of the VAR residuals for that variable, capturing the size of a typical innovation to the variable.

[22]The interpretation of our evidence is consistent with Ito et al. (2012)'s finding that in the foreign exchange market the probability of triangular arbitrage opportunities disappearing within a second has increased over time.

[23]The random walk, or martingale hypothesis, was traditionally linked to the efficient market hypothesis (e.g., Samuelson (1965), and Fama (1965, 1970), among others) —implying that efficient prices follow martingale processes and that returns exhibit no serial correlation, either positive (momentum) or negative (mean-reversion). We now know that predictability in prices is not inconsistent with rational equilibria and price efficiency when risk premia are changing. However it is highly unlikely that predictability in five-second returns reflects changing risk premia, and it seems reasonable to view high-frequency autocorrelation as an indicator of market (in-)efficiency.

[24]Our choice of a five-second frequency of returns is driven by a trade-off between sampling at a high enough frequency to estimate the effect algorithmic traders have on prices, but low enough to have sufficiently many transactions to avoid biasing the auto-correlations towards zero due to a high number of consecutive zero returns. Our results are qualitatively similar when we sample prices at the 1-second frequency for the most liquid currency pairs, euro-dollar and dollar-yen, but for the euro-yen the 1-second sampling frequency biases the return autocorrelations towards zero due to lower activity at the beginning of the sample.

[25]Our choice of estimating the autocorrelation over a five-minute interval is driven by a trade-off between a large enough window with sufficient observations to estimate the autocorrelation (12 observations in a one-minute window is too few observations) and small enough so that the Granger causality specification is sensible. We also perform a 99% Winsorising of the absolute values of the

autocorrelation estimates (that is, setting the largest 1% of the absolute autocorrelations equal to the 99th percentile of the absolute estimates).

[26]As discussed in Appendix C, there appears to be weak or no contemporaneous identification in the specification with $\ln{(R)}$ for the euro-yen currency pairs. All other specifications appear well identified.

[27]There is a very high correlation in this market between trading volume per unit of time and the number of transactions per unit of time, and the ratio between the two does not vary much over our sample. Order flow measures based on amounts transacted and those based on number of trades are therefore very similar.

[28]An instrumental variable is generally referred to as valid if it is exogenous and as relevant if it is correlated with the endogenous regressor.

Table I

Correlation Among AT Strategies: R-Measure

The table reports estimates of the relative degree to which computers trade with humans, based on the benchmark model described in the text. In particular, we report mean estimates of $\ln(R)$, $\ln(R^S)$, and $\ln(R^B)$, measured at the one-minute, five-minute, and daily frequencies, with standard errors shown in parentheses below the estimates. $\ln(R)$, $\ln(R^S)$, $\ln(R^B) > 0$ $(R, R^S, R^B > 1)$ indicates that computers trade less with each other than random matching would predict. The fraction of observations where $\ln(R)$, $\ln(R^S)$, $\ln(R^B) > 0$ is also reported along with the number of nonmissing observations and the total number of observations at each frequency. ***, **, and * represent a statistically significant deviation from zero at the 1%, 5%, and 10% level, respectively. The sample period is September 2003 to December 2007.

	one-minute data			five-minute data			Daily data		
	$\ln(R)$	$\ln(R^S)$	$\ln(R^B)$	$\ln(R)$	$\ln(R^S)$	$\ln(R^B)$	$\ln(R)$	$\ln(R^S)$	$\ln(R^B)$
					USD/EUR				
Mean	0.2219***	0.0544***	0.0401***	0.3674***	0.2118***	0.2078***	0.5314***	0.4901***	0.4995***
(standard error)	(0.0031)	(0.0042)	(0.0042)	(0.0036)	(0.0046)	(0.0047)	(0.0118)	(0.0122)	(0.0118)
Fraction of observations greater than zero	0.599	0.527	0.522	0.723	0.627	0.626	0.990	0.975	0.974
Number of nonmissing observations	143421	89914	87555	52101	44310	43560	880	846	854
Total number of observations	512160	512160	512160	102432	102432	102432	1067	1067	1067
					JPY/USD				
Mean	0.3106***	0.1536***	0.1515***	0.3925***	0.2120***	0.2113***	0.5849***	0.5758***	0.5401***
(standard error)	(0.0037)	(0.0052)	(0.0052)	(0.0043)	(0.0054)	(0.0054)	(0.0131)	(0.0144)	(0.0134)
Fraction of observations greater than zero	0.611	0.545	0.545	0.703	0.609	0.610	0.990	0.969	0.971
Number of nonmissing observations	114449	61023	61645	49906	39036	39362	953	938	922
Total number of observations	512160	512160	512160	102432	102432	102432	1067	1067	1067
					JPY/EUR				
Mean	0.6987***	0.5310***	0.5198***	0.6873***	0.5842***	0.5838***	0.8130***	0.7733***	0.7380***
(standard error)	(0.0049)	(0.0074)	(0.0076)	(0.0051)	(0.0066)	(0.0068)	(0.0173)	(0.0171)	(0.0164)
Fraction of observations greater than zero	0.696	0.643	0.636	0.758	0.700	0.698	0.984	0.975	0.965
Number of nonmissing observations	71783	30567	29337	45846	31629	31010	987	951	942
Total number of observations	512160	512160	512160	102432	102432	102432	1067	1067	1067

Table II

Triangular Arbitrage and AT Activity

We report tests of whether AT activity has a causal impact on triangular arbitrage (Panel A) and whether triangular arbitrage has a causal impact on AT activity (Panel B). Results are presented separately for each of the following measures of algorithmic trading activity averaged across currency pairs: overall AT participation (VAT), AT taking participation (VCt), AT making participation (VCm), relative computer taking activity ($OFCt$), and the natural logarithm of AT trade correlation ($\ln(R)$). All results are based on one-minute data covering the full sample period from 2003 to 2007. The first five rows in each panel present the results from Granger causality tests, based on the reduced-form VAR in equation (2). In particular, the first three rows in each panel report the sum of the lag-coefficients for the causing variable, and the corresponding Wald χ^2-statistic and p-value for the null hypothesis that the sum of the coefficients is equal to zero. The fourth and fifth rows report the Wald χ^2-statistic and p-value for the null hypothesis that the coefficients on all lags of the causing variable are jointly equal to zero. The following row, labeled contemporaneous coefficient, presents the point estimate of the contemporaneous impact of the causing variable in the structural VAR in equation (1), based on the heteroskedasticity identification scheme described in the text, with the Newey–West standard error presented below in parentheses. The contemporaneous impact scaled by the residual standard deviation, as a measure of economic magnitude, is reported in the second-to-last row in each panel. The last row in each panel shows the total number of observations available for estimation in the full sample. ***, **, *, and * represent a statistically significant deviation from zero at the 1%, 5%, and 10% level, respectively.

|	VAT	VCt	VCm	$OFCt$	$\ln(R)$
Panel A. Tests of AT Causing Triangular Arbitrage					
Sum of coefficients on AT lags	−0.0027***	−0.0061***	0.0049***	−0.0117***	−0.1202***
χ^2_1 (Sum = 0)	7.7079***	25.2501***	14.4351***	142.4661***	15.3976***
p-value	0.0055	0.0000	0.0001	0.0000	0.0001
χ^2_{20} (All coefficients on AT lags = 0)	91.4066***	149.5884***	46.5108***	317.3994***	40.9929***
p-value	0.0000	0.0000	0.0007	0.0000	0.0037
Contemporaneous coefficient	−0.0145***	−0.0067**	−0.0040***	−0.0288***	−1.4665**
(standard error)	(0.0040)	(0.0027)	(0.0011)	(0.0055)	(0.7025)
Contemporaneous coefficient×σ_{AT}	−0.1932***	−0.0813**	−0.0406***	−0.4813***	−1.5193**
Number of observations	512016	512016	512016	511688	176713
Panel B. Tests of Triangular Arbitrage Causing					
Sum of coefficients on arb lags	0.0140***	0.0219***	−0.0022	0.0345***	0.0005
χ^2_1 (Sum = 0)	19.5814***	56.9040***	0.8233	75.9510***	1.6980
p-value	0.0000	0.0000	0.3642	0.0000	0.1925
χ^2_{20} (All coefficients on arb lags = 0)	83.0479***	118.9288***	31.0039*	459.3056***	84.9251***
p-value	0.0000	0.0000	0.0551	0.0000	0.0000
Contemporaneous coefficient	0.6072***	0.4817***	0.0545***	1.3037***	0.1598***
(standard error)	(0.0591)	(0.0331)	(0.0089)	(0.1241)	(0.0617)
Contemporaneous coefficient×σ_{arb}	2.0405***	1.6185***	0.1832***	4.3776***	0.5308***
Number of observations	512016	512016	512016	511688	176713

Table III

Autocorrelation of High-Frequency Returns and AT Activity

We report tests of whether AT activity has a causal impact on autocorrelation (Panels A, C, E, G and I) and whether autocorrelation has a causal impact on AT activity (Panels B, D, F, H and J). Results are presented separately for each currency pair and for each of the following measures of algorithmic trading activity averaged across currency pairs: overall AT participation (VAT), AT taking participation (VCt), AT making participation (VCm), relative computer taking activity ($OFCt$), and the natural logarithm of AT trade correlation ($\ln(R)$). All results are based on one-minute data covering the full sample period from 2003 to 2007. The first five rows in each panel present the results from Granger causality tests, based on the reduced-form VAR in equation (4). In particular, the first three rows in each panel report the sum of the lag-coefficients for the causing variable, and the corresponding Wald χ^2-statistic and p-value for the null hypothesis that the sum of the coefficients is equal to zero. The fourth and fifth rows report the Wald χ^2-statistic and p-value for the null hypothesis that the coefficients on all lags of the causing variable are jointly equal to zero. The following row, labeled contemporaneous coefficient, presents the point estimate of the contemporaneous impact of the causing variable in the structural VAR in equation (3), based on the heteroskedasticity identification scheme described in the text, with the Newey-West standard error presented below in parentheses. The contemporaneous impact scaled by the residual standard deviation, as a measure of economic magnitude, is reported in the second-to-last row in each panel. The last row in each panel shows the total number of observations available for estimation in the full sample. ***, **, and * represent a statistically significant deviation from zero at the 1%, 5%, and 10% level, respectively.

	USD/EUR	JPY/USD	JPY/EUR
Panel A. Tests of VAT Causing Autocorrelation			
Sum of coefficients on VAT lags	−0.03904***	−0.02562***	−0.01539***
χ^2_1 (Sum = 0)	33.7507***	25.2649***	14.9665***
p-value	0.0000	0.0000	0.0001
χ^2_{20} (All coefficients on VAT lags = 0)	43.0591***	45.5455***	15.1881***
p-value	0.0000	0.0000	0.0043
Contemporaneous coefficient	−0.04770***	−0.02377***	−0.03274***
(standard error)	(0.00703)	(0.00622)	(0.00927)
Contemporaneous coefficient×σ_{VAT}	−0.34388***	−0.24414***	−0.50116***
Number of observations	102432	102427	102113
Panel C. Tests of VCt Causing Autocorrelation			
Sum of coefficients on VCt lags	−0.04325***	−0.02187***	−0.01069**
χ^2_1 (Sum = 0)	23.0192***	11.8863***	6.1195*
p-value	0.0000	0.0006	0.0134
χ^2_{20} (All coefficients on VCt lags = 0)	24.7961***	20.8758***	6.6594
p-value	0.0001	0.0003	0.1550
Contemporaneous coefficient	−0.02630***	−0.00577	−0.00255
(standard error)	(0.00752)	(0.00559)	(0.00567)
Contemporaneous coefficient×σ_{VCt}	−0.15686***	−0.05066	−0.03696
Number of observations	102432	102427	102113

	USD/EUR	JPY/USD	JPY/EUR
Panel B. Tests of Autocorrelation Causing VAT			
Sum of coefficients on ac lags	−0.00689*	−0.01029*	−0.02080***
χ^2_1 (Sum = 0)	3.0155**	3.4939**	7.8843***
p-value	0.0825	0.0616	0.0050
χ^2_{20} (All coefficients on ac lags = 0)	12.8762***	4.0260	12.2112**
p-value	0.0119	0.4025	0.0158
Contemporaneous coefficient	0.00330*	0.00100	0.04918***
(standard error)	(0.00188)	(0.00427)	(0.01428)
Contemporaneous coefficient×σ_{ac}	0.03607*	0.01129	0.60821***
Number of observations	102432	102427	102113
Panel D. Tests of Autocorrelation Causing VCt			
Sum of coefficients on ac lags	−0.00019	−0.00123	−0.00128
χ^2_1 (Sum = 0)	0.0033	0.0688	0.0333
p-value	0.9545	0.7931	0.8553
χ^2_{20} (All coefficients on ac lags = 0)	2.6777	1.7370	4.8289
p-value	0.6131	0.7840	0.3053
Contemporaneous coefficient	0.00138	−0.00343	−0.00741
(standard error)	(0.00105)	(0.00247)	(0.00728)
Contemporaneous coefficient×σ_{ac}	0.01509	−0.03875	−0.09160
Number of observations	102432	102427	102113

Table III

Autocorrelation of High-Frequency Returns and AT Activity (continued)

Panel E. Tests of VCm Causing Autocorrelation

	USD/EUR	JPY/USD	JPY/EUR
Sum of coefficients on VCm lags	−0.04692***	−0.03620***	−0.02146***
χ^2_1 (Sum = 0)	29.7277***	28.8421***	17.8840***
p-value	0.0000	0.0000	0.0000
χ^2_{20} (All coefficients on VCm lags = 0)	44.1685***	38.8638***	18.9016***
p-value	0.0000	0.0000	0.0008
Contemporaneous coefficient	−0.07073***	−0.03907***	−0.02409***
(standard error)	(0.00902)	(0.00711)	(0.00622)
Contemporaneous coefficient$\times\sigma_{VCm}$	−0.38444***	−0.31120***	−0.29510***
Number of observations	102432	102427	102113

Panel F. Tests of Autocorrelation Causing VCm

	USD/EUR	JPY/USD	JPY/EUR
Sum of coefficients on ac lags	−0.00917***	−0.01358***	−0.02662***
χ^2_1 (Sum = 0)	9.3691	10.1188	20.1782
p-value	0.0022	0.0015	0.0000
χ^2_{20} (All coefficients on ac lags = 0)	33.8031***	11.0068**	20.2980***
p-value	0.0000	0.0265	0.0004
Contemporaneous coefficient	0.00156	0.00268	0.03122***
(standard error)	(0.00137)	(0.00269)	(0.00571)
Contemporaneous coefficient$\times\sigma_{ac}$	0.01707	0.03027	0.38603***
Number of observations	102432	102427	102113

Panel G. Tests of $OFCt$ Causing Autocorrelation

	USD/EUR	JPY/USD	JPY/EUR
Sum of coefficients on $OFCt$ lags	−0.00481	−0.00154	0.00013
χ^2_1 (Sum = 0)	2.3004	0.2637	0.0021
p-value	0.1293	0.6076	0.9638
χ^2_{20} (All coefficients on $OFCt$ lags = 0)	2.8266	1.8544	1.3831
p-value	0.5872	0.7625	0.8471
Contemporaneous coefficient	0.00434	−0.00631	−0.00124
(standard error)	(0.00455)	(0.00463)	(0.00464)
Contemporaneous coefficient$\times\sigma_{OFCt}$	0.09057	−0.14479	−0.03253
Number of observations	102250	101992	100815

Panel H. Tests of Autocorrelation Causing $OFCt$

	USD/EUR	JPY/USD	JPY/EUR
Sum of coefficients on ac lags	−0.02571**	−0.01388	−0.01343
χ^2_1 (Sum = 0)	5.0050**	1.2702	1.0988
p-value	0.0253	0.2597	0.2945
χ^2_{20} (All coefficients on ac lags = 0)	8.5794*	4.7680	2.7475
p-value	0.0725	0.3119	0.6009
Contemporaneous coefficient	−0.00958	0.03736**	0.01716
(standard error)	(0.01482)	(0.01857)	(0.02067)
Contemporaneous coefficient$\times\sigma_{ac}$	−0.10473	0.42178**	0.21192
Number of observations	102250	101992	100815

Panel I. Tests of $\ln(R)$ Causing Autocorrelation

	USD/EUR	JPY/USD	JPY/EUR
Sum of coefficients on $\ln(R)$ lags	−0.03022	0.00984	0.06147
χ^2_1 (Sum = 0)	0.06553	0.00931	0.40488
p-value	0.79795	0.92312	0.52458
χ^2_{20} (All coefficients on $\ln(R)$ lags = 0)	3.09457	2.66549	0.96692
p-value	0.54213	0.61527	0.91477
Contemporaneous coefficient	−0.01286	−0.01857	0.00017
(standard error)	(0.30686)	(0.49029)	(13.18378)
Contemporaneous coefficient$\times\sigma_{\ln(R)}$	−0.00978	−0.01656	0.00017
Number of observations	50837	48266	44225

Panel J. Tests of Autocorrelation Causing $\ln(R)$

|	USD/EUR	JPY/USD	JPY/EUR
Sum of coefficients on ac lags	0.00056	−0.00050	−0.00205**
χ^2_1 (Sum = 0)	0.91886	0.46341	5.63208
p-value	0.33778	0.49604	0.01763
χ^2_{20} (All coefficients on ac lags = 0)	5.05363	1.69574	7.25750
p-value	0.28184	0.79149	0.12289
Contemporaneous coefficient	−0.00058	−0.00066	−0.00060
(standard error)	(0.00146)	(0.00340)	(0.11685)
Contemporaneous coefficient$\times\sigma_{ac}$	−0.00643	−0.00706	−0.00661
Number of observations	50837	48266	44225
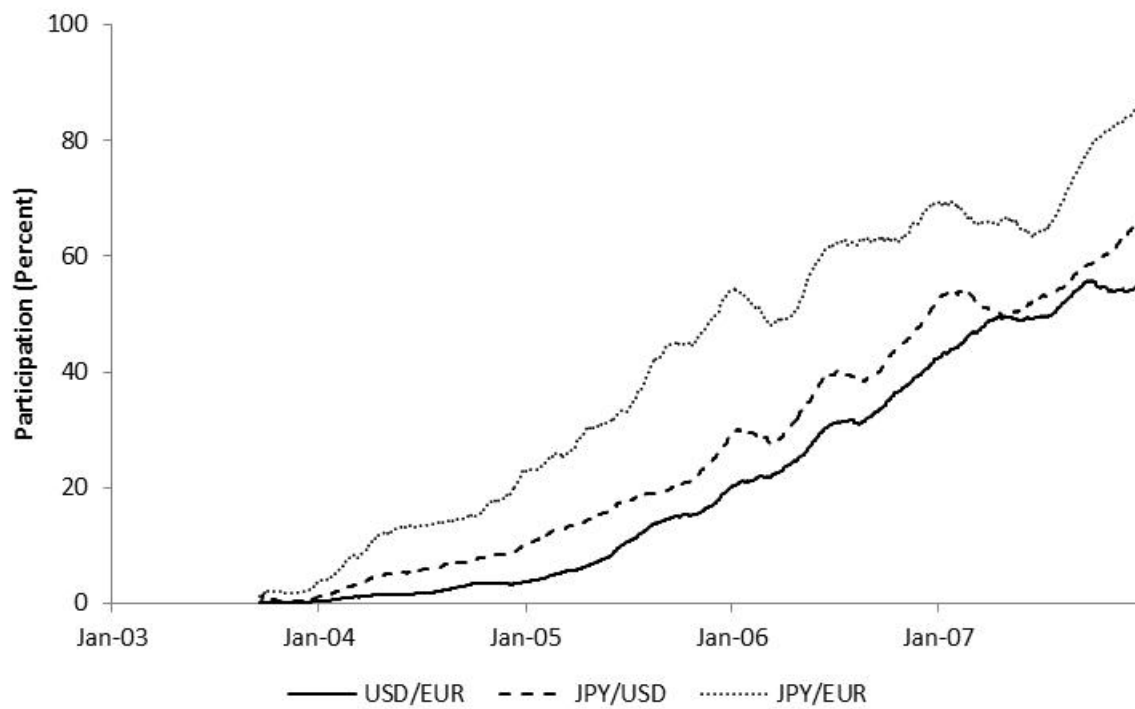
Figure 1. 50-day moving averages of VAT, the percent of total volume with at least one algorithmic counterparty.
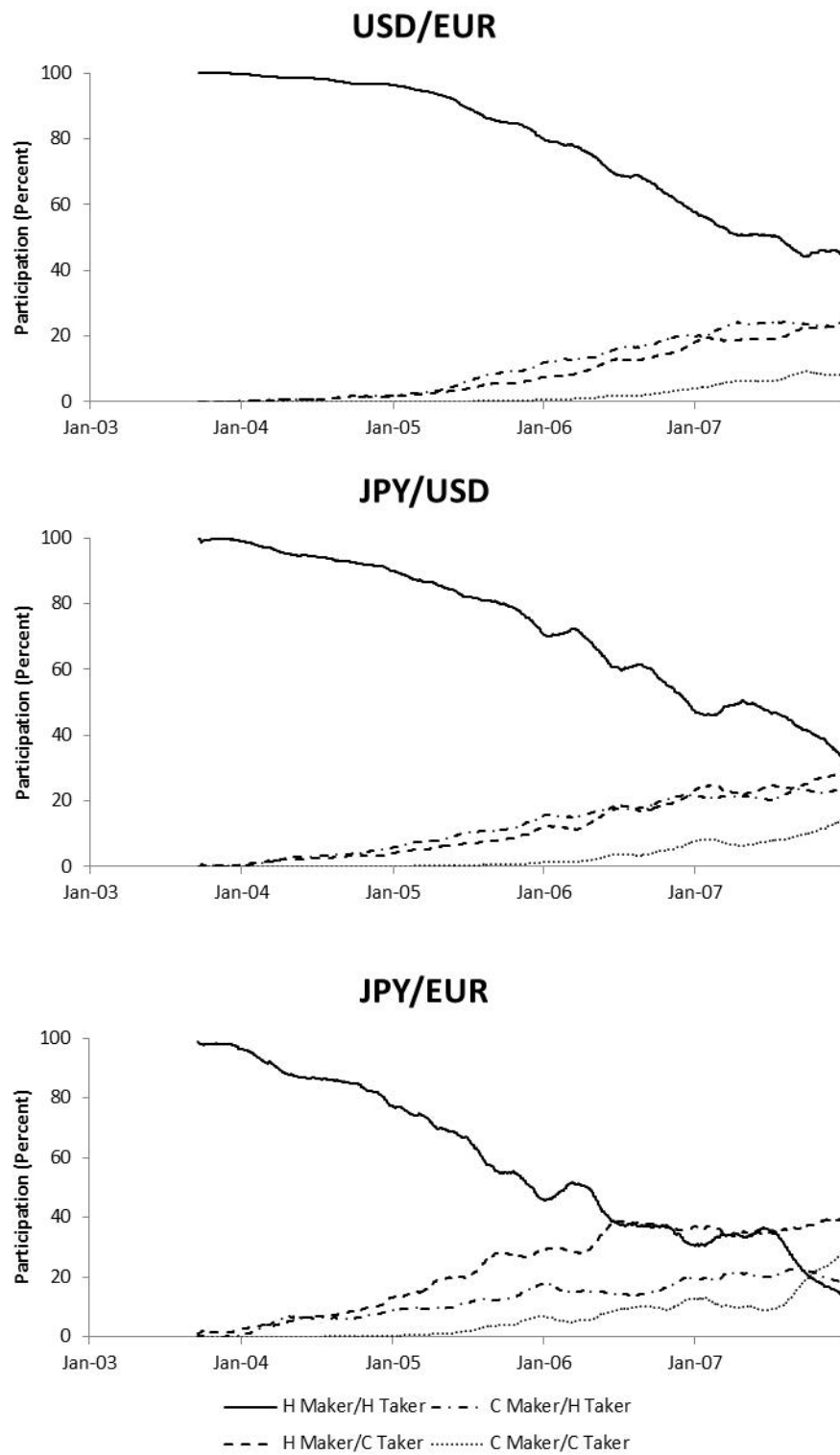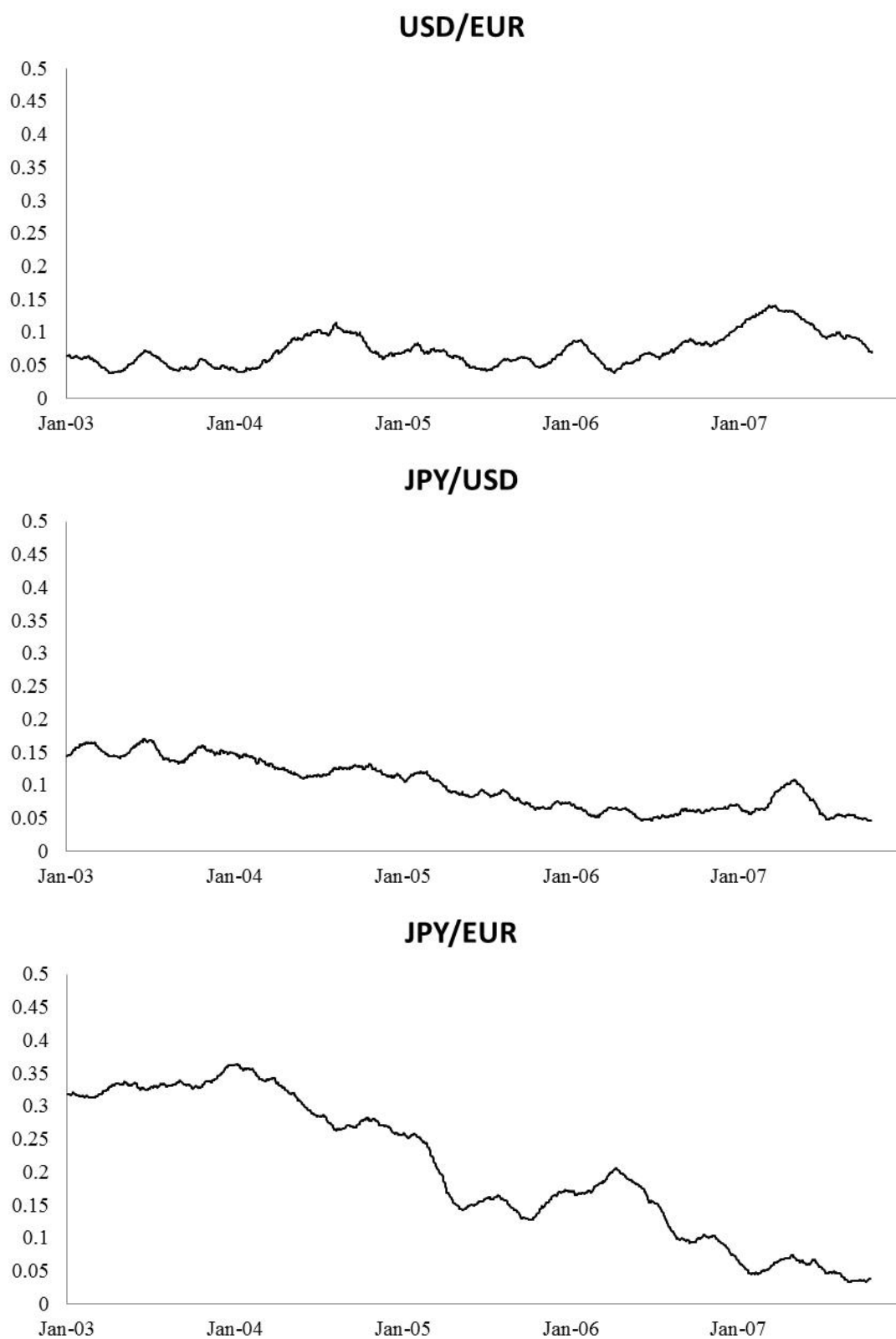
Figure 2. 50-day moving averages of the percent of total volume broken down into four maker-taker pairs
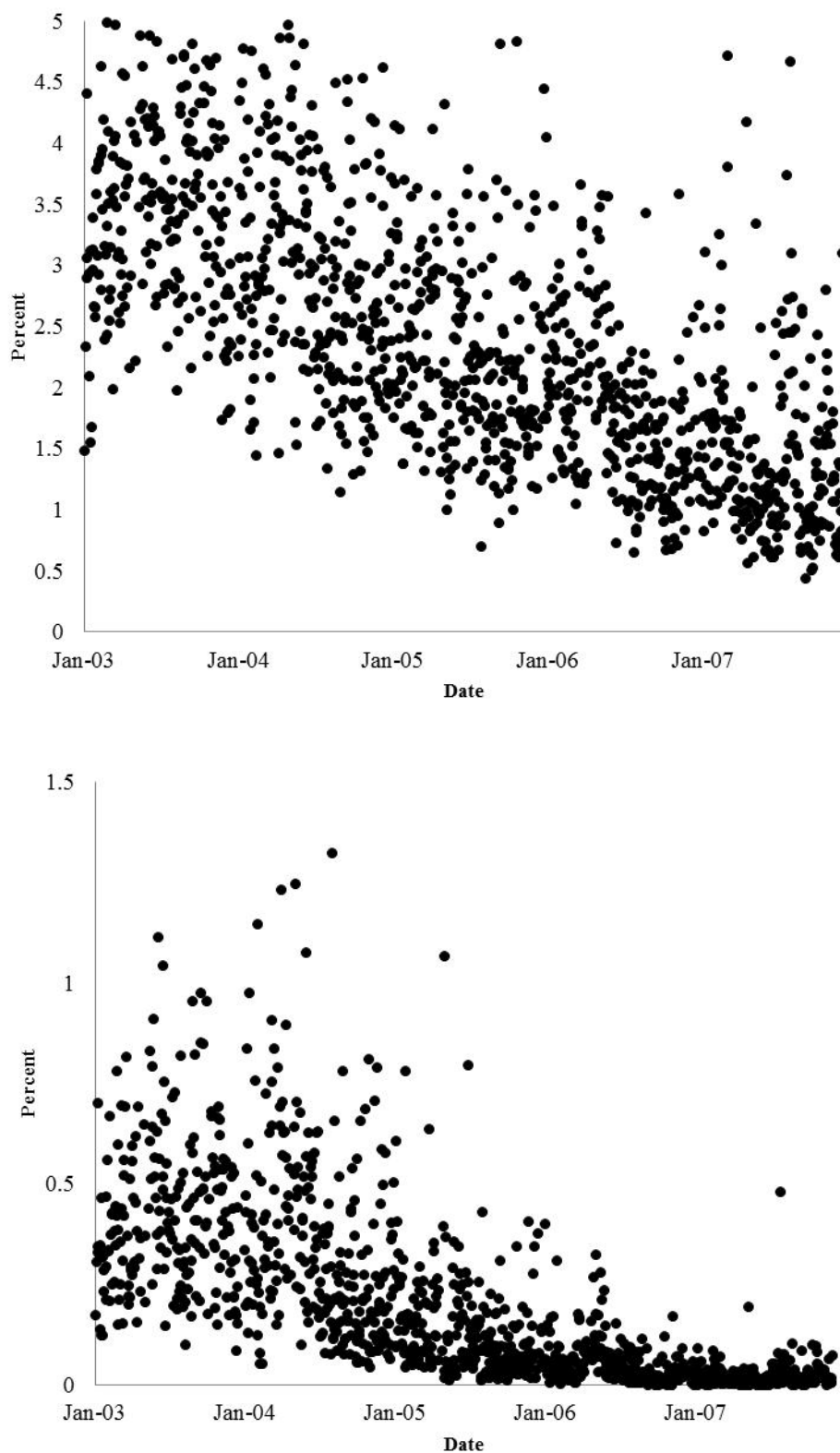
Figure 3. Percent of seconds with a triangular arbitrage opportunity with a profit strictly greater than zero basis point (top panel) and 1 basis point (bottom panel).

Figure 4: 50-day moving averages of absolute value of 5-second return serial autocorrelation estimated each day using observations from 3:00 am ET to 11:00 am ET.