CONFIDENTIAL

Analysis and Development Of Correlation Arbitrage Strategies on Equities

-Report on internship at ***Lyxor Asset Management***-

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SUMMARY (ENGLISH)

After the two years of studies in the area of mathematical finance at University of Paris 1, I had a chance to work with an asset management team as a quantitative analyst at Lyxor Asset Management, Société Générale in Paris, France.

My first task was to develop an analysis of the performances of the funds on “hidden assets” where the team’s main focus was on, such as Volatility Swap, Variance Swap, Correlation Swap, Covariance Swap, Absolute Dispersion, Call on Absolute Dispersion (Palladium). The purpose was to anticipate the profit and to know when and how to reallocate assets according to the market conditions. In particular, I have automated the analysis through VBA in Excel.

Secondly, I had a research project on Correlation trades especially involving Correlation Swaps and Dispersion Trades. This report is to summarize the research I have conducted in this subject. Lyxor has been benefiting from taking short positions on Dispersion Trades through variance swaps, thanks to the fact that empirically the index variance trades ‘rich’ with respect to the variance of the components. However, a short position on a dispersion trade being equivalent to taking a long position in correlation, in case of a market crash (or a volatility spike), we can have a loss in the position. Thus, the goal of the research was to find an effective hedging strategy that can protect the fund under unfavorable market conditions. The main idea was to apply the fact that dispersion trades and correlation swaps are both ways to have exposure on correlation, but with different risk factors. While correlation swap has a ‘pure’ exposure to correlation, dispersion trade has exposure to the realised volatilities as well as the correlation of the components. Thus, having risk to another factor, the implied correlation of a dispersion trade is above (empirically, 10 points) the strike of the equivalent correlation swap. Thus, taking these two products and taking opposite positions in the two, we try to achieve a hedging effect. Furthermore, I look for the optimal weight of the two products in the strategy which gives us the return of the P&L, volatility of the P&L, and risk-return ratio of our preference. Moreover, I tested how this strategy would have performed in past market conditions (back-test) and under extremely bearish market conditions (stress-test).

After the research project, for the remaining period of my internship, I will conduct back-test, stress-test and sensitivity-test on new strategies developed.

SUMMARY (FRANCAISE)

Après deux ans d'études en mathématiques financières, j'ai eu une grande opportunité d'appliquer mes connaissances dans ce domaine comme analyste quantitatif, avec l'équipe des gérants de fonds, au sein de Lyxor Asset Management, Filiale de la Société Générale, La Défense, Paris.

La première étape de mon stage fut de développer une analyse de performance pour des fonds, ayant des stratégies basées sur des paramètres implicites. Les fonds gérés par mon équipe sont principalement basés sur des swaps de volatilité, swaps de Variance, swaps de Covariance, Dispersion Absolue, Call sur Dispersion Absolue (Palladium). L'objectif étant d'anticiper les profits et de déterminer la meilleure partition des actifs du portefeuille en fonction l'état du marché. Pour ce faire, j'ai développé des macros sous Excel afin de faire des tests sur des données historiques.

En deuxième étape, j'ai effectué des recherches sur les contrats de Corrélation, en particulier les Swaps de Corrélation et les contrats de Dispersion. Ce rapport développe essentiellement les recherches que j'ai conduite au cours de cette deuxième étape. La stratégie qui a une position Short sur des contrats de dispersion à travers des Swaps Variance fut profitable ces dernières années car la variance des indices ont été globalement supérieurs à ceux du panier correspondant. Cependant, puisque une Position Short sur un contrat de Dispersion est équivalent à prendre une position Long sur Corrélation, si un 'Crash' boursier survient, ou un comportement de volatilité équivalent, on peut avoir de grandes pertes liées à cette position. C'est pour se couvrir face à un tel comportement du marché, que toute cette étude prend un sens. Il s'agit de développer une stratégie qui permettra au fonds de se protéger en cas de comportement défavorable du marché. L'idée principale étant d'utiliser le fait que les contrats de Dispersion et les Swaps de Corrélation permettent les deux d'avoir une exposition sur la corrélation, mais avec différents facteurs de risque. Alors que le Swap de Corrélation est une exposition 'pure' à la corrélation, un contrat de Dispersion est exposé à l'ensemble de la volatilité réalisée du panier et de la corrélation. Etant exposée sur un autre facteur, le Strike d'une corrélation implicite d'un contrat de Dispersion est au dessus du Strike d'un Swap de corrélation équivalent. Donc, en considérant ces deux produits financiers et en adoptant des positions différentes sur les deux produits dans la stratégie, on essaie d'établir une couverture. En plus, il s'agit d'optimiser la moyenne du P&L, la volatilité du P&L et le ratio de retour sur risque en paramétrant les poids des deux produits (Swap de Corrél et contrat de Dispersion). J'ai également testé les performances de cette stratégie sur des données historiques, et comment elle aurait performé dans le passé (Back-test) et sous des conditions du marché extrêmement défavorables (Stress-test).

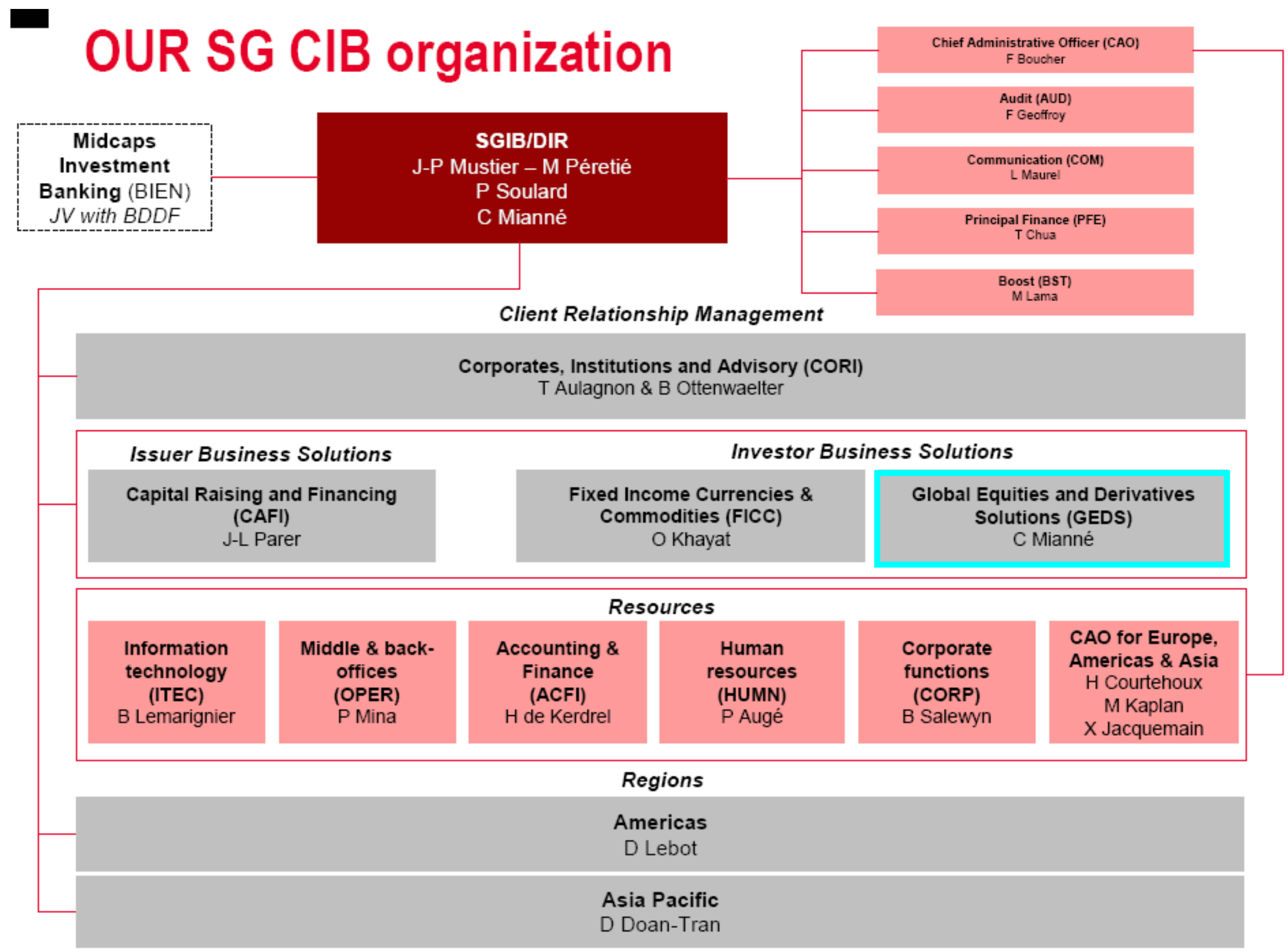
Après avoir accomplis mes recherches, pour le temps restant de mon stage, je vais accomplir des back-test, stress-test et des test de sensibilité pour de nouvelles stratégies.

INDEX

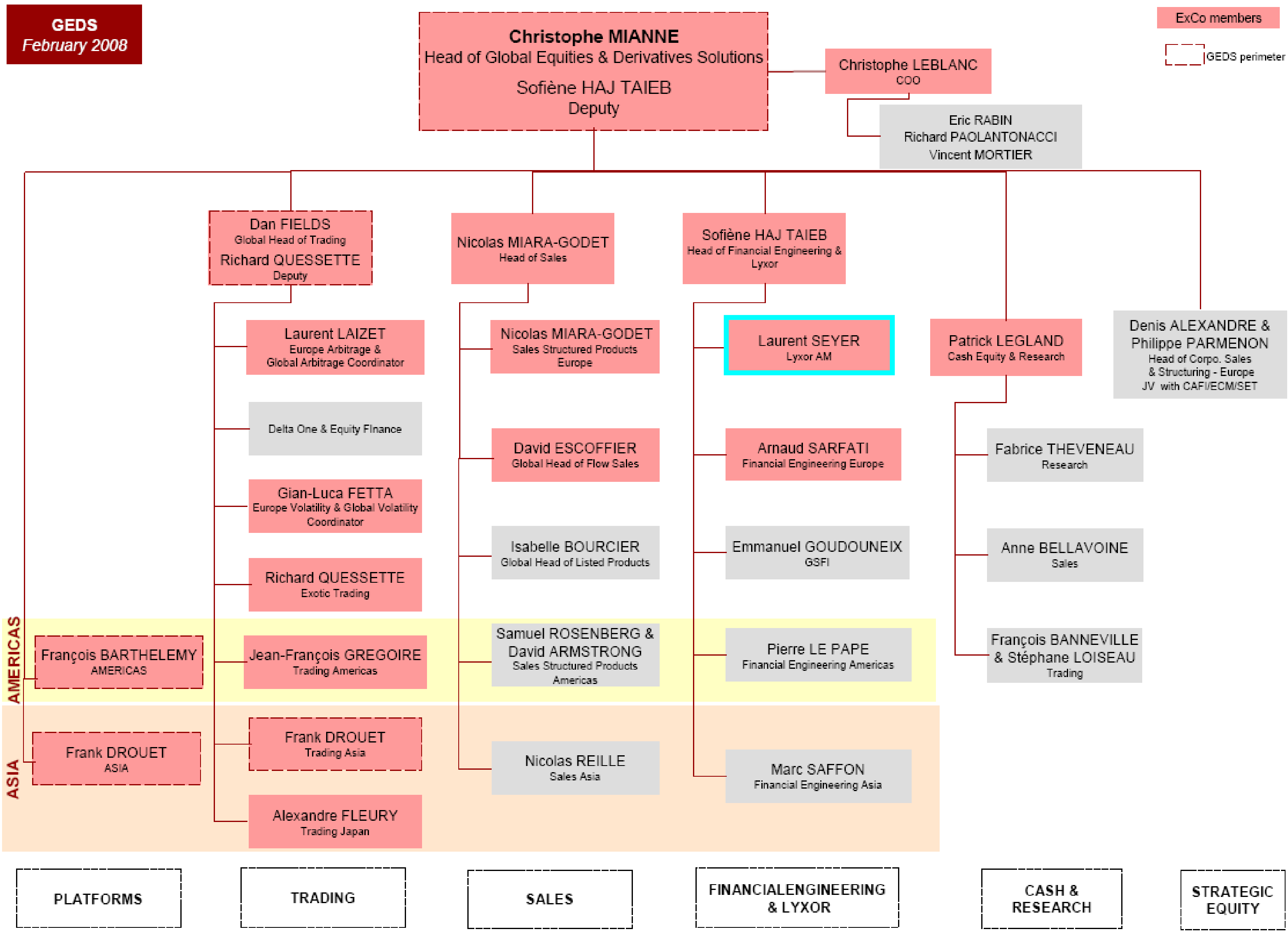
LYXOR ASSET MANAGEMENT

Lyxor Asset Management was created in 1998 by the Global Equities and Derivatives Solutions (GEDS), the Financial Engineering Department of SOCIETE GENERALE Corporate and Investment Banking (SGCIB). Under are the organization charts to demonstrate where Lyxor stands in SOCIETE GENERALE.

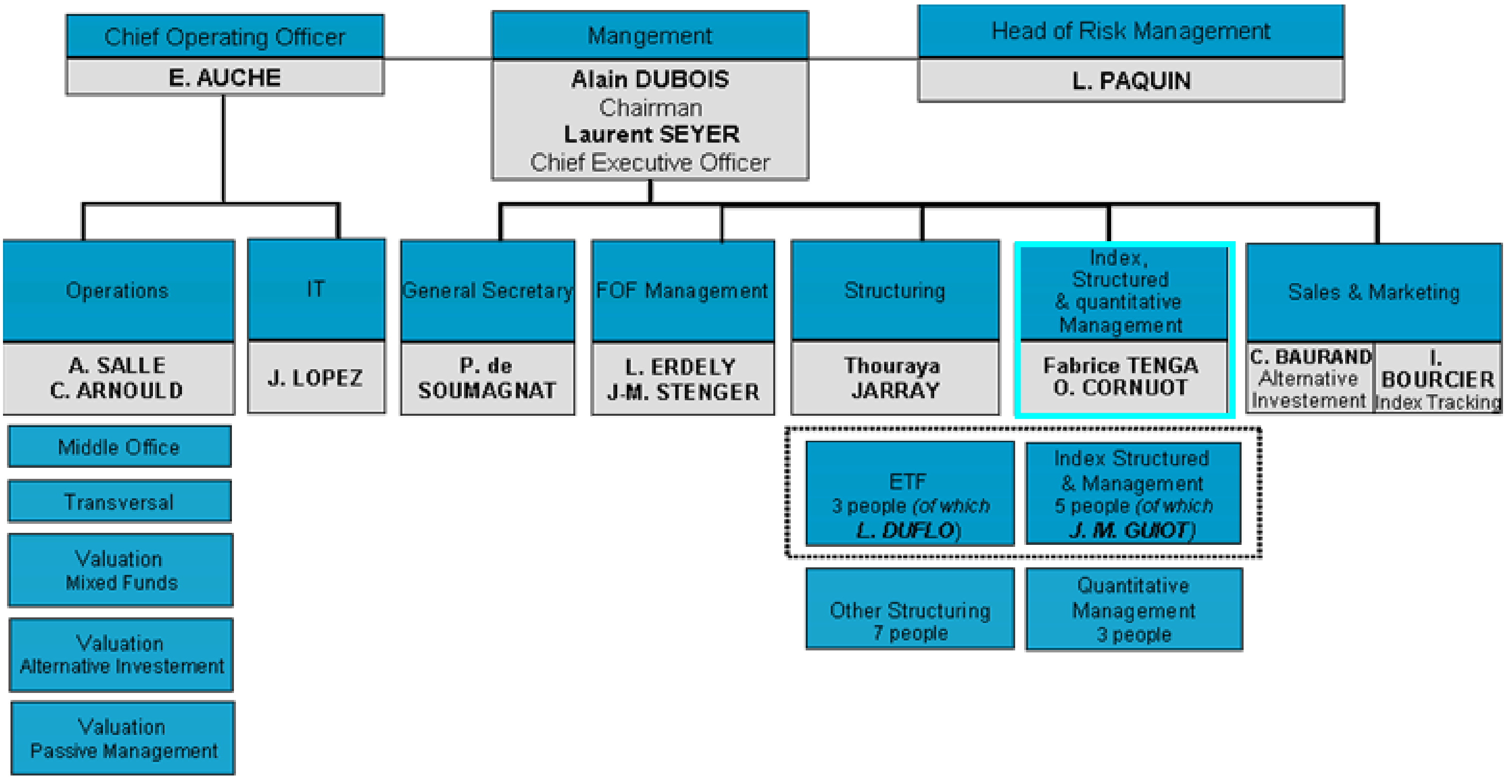
ORGANIZATION CHART OF SG CIB



ORGANIZATION CHART OF GEDS



ORGANIZATION CHART OF LYXOR



PRODUCTS

Lyxor Asset Management has positioned itself as a specialist, a niche player, with expertise on three expanding investment market.

- Index tracking : 1012 funds
- Structured funds: 394 funds
- Alternative investments: 127 funds

The amount of Asset under management is:

Total AUM:	€ 75.2 billion	100%
Index Tracking	€ 28.90 billion	38.4%
Structured Investment	€ 20.50 billion	27.3%
Alternative Investment	€ 25.80 billion	34.3%

Source: Lyxor Asset Management (figures as of May 27th, 2008)

Among these funds, my team managed the structured funds. In particular, they were categorized as Lyxor Generis fund and Lyxor Quantic fund.

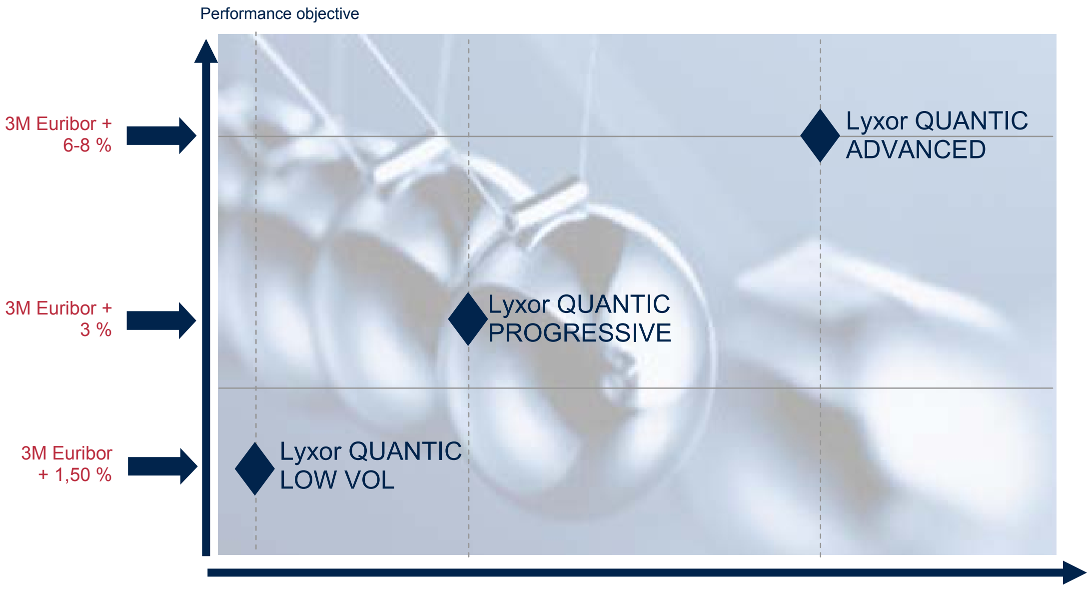
GENERIS

Lyxor Generis is consisted of pure single-strategy hedge funds. It takes advantage from inefficiencies on equity implied assets thus takes statistical arbitrage through a model-based process. It consists of 4 strategies under the names Lyxor G-Volt, Lyxor G-Square, Lyxor G-Sphere, Lyxor G-Smile, and the main ideas for each funds are described in the following chart.

	Direct exposure to implied assets			
	Lyxor G-Volt	**Lyxor G-Square**	**Lyxor G-Sphere**	**Lyxor G-Smile**
	■ **Stock Volatility Arbitrage Strategy**	■ **Index Volatility Surface Arbitrage Strategy**	■ **Index Implied/Realized Volatility Arbitrage Strategy**	■ **Volatility Smile Arbitrage Strategy**
Core beliefs	■ Volatility is an asset ■ Market are not efficient when pricing volatility	■ Main indices volatilities are correlated on a short term	■ Long volatility position hedged against market reversal	■ Asymmetric investor behaviour towards buy and sell recommendation
Inefficiency	■ The risk of certain stocks is undervalued with respect to their market	■ Markets price a non-convergence of indices volatility	■ In normal market conditions, implied volatility is priced above the realised volatility level	■ Non realisation of stock smile
Strategy	Arbitrage between equity and indices volatilities ■ Long stocks volatility ■ Short indices volatility	■ Arbitrage between implied volatility forward ■ Arbitrage between implied volatility smile	■ Short spot index volatility ■ Long position on implied volatility	■ Volatility Smile Arbitrage ■ Short stock smile ■ Long stock volatility
Objectives	■ Performance: 15% p.a. ■ Volatility: 10% max. p.a.	■ Performance: 15% p.a. ■ Volatility: 10% max. p.a.	■ Performance: 15% p.a. ■ Volatility: 10% max. p.a.	■ Performance: 15% p.a. ■ Volatility: 10% max. p.a.

QUANTIC

Lyxor Quantic is a more active and sophisticated investment strategy compared to Lyxor Generis. Still the main idea is to take advantages of inefficiencies on implied assets. It consists of four strategies named Lyxor Low Vol, Lyxor Progressive, Lyxor Dynamic, which differs in the performance objective.



And the different performances are achieved through different strategies.

Types of Strategy	Drivers	Average Maturity	Example of positions
Discretionary strategy	Fundamental analysis Quantitative Inputs	From 3 months to 3 years	Relative value and opportunistic position on implied assets
Quantitative arbitrage	Quantitative models Discretionary active allocation	2 years	Statistical arbitrage and systematic arbitrage on volatility, dispersion, correlation, smile…
Tactical overlay	Short term views	Below 3 months	Directional positions on volatility , short term indices option (vanilla option or barrier option)

During my internship, most of my tasks were dedicated to Lyxor Quantic.

TEAM MEMBERS

I worked with the asset management team of 6 people including 4 fund managers 1 quantitative analyst and 1 intern. Following is a brief description of the team members.

Fabrice Tenga – Head of Fund Management Team

- Fabrice joined Société Générale as a Financial Engineer in charge of Pricing and New Product Creation for European market. In 2002, he moved to Hong Kong to head the equity derivatives financial engineering team in charge of Asia ex Japan. In 2005 he became head of the financial engineering team based in Milan in charge of the Italian market before being appointed Head of Fund Management Team of Lyxor in 2007. Fabrice is a graduate from ENSAE in Malakoff, one of the leading french grandes ecoles specialized in economics, statistics, probability and finance and has a Master degree in Statistics, economic modelisation and Finance from the University of Paris VII.

Olivier Cornuot – Deputy head of Fund Management Team, responsible for Lyxor Quantic Funds

- Olivier joined Lyxor AM in 2006 to take charge of the Lyxor Quantic management team. Previously, Olivier was a portfolio manager at BNP PAM, first as a global balanced portfolio manager, then as a member of the structured asset management team. He managed structured funds (mainly guaranteed or protected accounts) and also contributed to the development of the activity through the development and management of innovative structured funds. He started his career at Paribas' Internal Audit Department as an Auditor specialized in Capital Markets and Asset Management. He is a graduate of the French engineering school "Ecole Nationale des Ponts et Chaussées" as well as of the "Institut d'Etudes Politiques" in Paris. He is a CFA charter holder.

Waiel Ben-Gaied – fund manager

- Waiel is a fund manager of Lyxor Quantic funds. Waiel joined Lyxor AM in 2006 as a financial engineer in the Structured Management Team. He was appointed as fund manager in 2007. Previously, Waiel was a financial engineer in HSBC France. Waiel graduated from the French engineering school “Ecole Nationale des Ponts et Chaussées” and holds a Master in Probability and Finance

Quentin Perromat - fund manager

- Quentin joined Lyxor AM as a fund manager in 2007. Previously, he had worked in SOCIETE GENERALE where he was risk analyst on equity and index derivatives. Quentin graduated from the French engineering school “SUPAERO” financial engineering specializations.

Braihm Sentissi – Quantitative analyst

- Braihm joined Lyxor AM as a quantitative analyst in 2008. Braihm graduated from Ecole Centrale. Braihm had been working as a quantitative analyst in SOCIETE GENERALE for two years.

GROWING INTEREST IN CORRELATION (ρ)

In the recent years, banks have sold structured products such as Worst-of options[1], Everest[2] and Himalayas[3], resulting in a short correlation exposure. They have hence become interested in offsetting part of this exposure, namely buying back correlation. Two ways have been proposed for such a strategy: either pure correlation swaps or dispersion trades which consist in taking a position in an index option and the opposite position in the components options. These dispersion trades have been traded using calls, puts, straddles, and they now trade variance swaps as well as third generation volatility products, namely gamma swaps and barrier variance swaps. In this report we will focus on the dispersion trades through variance swaps.

To repeat, correlation exposure is achieved either through a Dispersion Trade or via Correlation Swap.

Correlation Swap gives a direct exposure to correlation; with the payoff of the difference between the realised correlation (the average Pairwise correlation) and the strike set at inception of the trade. Although the Correlation Swap is the most direct way to trade correlation, this instrument is not very liquid, since there is no methodology to estimate the Implied Pairwise Correlation, meaning the valuation of the fair strike and hedging of the instrument is very problematic. Therefore, the most common way to have a “direct” exposure to correlation is via Dispersion Trades.

[1] Option that pays the worst performance among stocks within a pre-specified basket
[2] Payoff on the worst performing
[3] Average performance of best share of index

When considering a dispersion trade via variance swaps, one immediately sees that it gives a correlation exposure independent of the level of the stocks. But it has empirically been showed that the implied correlation –in such a dispersion trade-was not equal to the strike of a correlation swap with the same maturity. Indeed, the implied correlation tends to be around 10 points higher. We will further discuss this issue, and we begin by explaining the definition of correlation used in this report.

DISPERSION & CORRELATION

DISPERSION

Dispersion is the difference between the average of single stocks volatility and the benchmark index volatility.

$$Disp = \sum_{i=1}^{N} \omega_i \sigma_i - \sigma_I$$

CORRELATION

Correlation is a measure of the tendency of share prices to move together. There are three different ways to calculate correlation.

- **AVERAGE PAIRWISE CORRELATION**

The correlation between two assets, or pairwise correlation between date 0 and T, is defined as:

$$\rho_{i,j}^{o,T} = \frac{\sum_{i=0}^{T} \left(r_{i,t} - \overline{r_i}\right)\left(r_{j,t} - \overline{r_j}\right)}{\sqrt{\sum_{i=0}^{T} \left(r_{i,t} - \overline{r_i}\right)^2} \sqrt{\sum_{i=0}^{T} \left(r_{j,t} - \overline{r_j}\right)^2}} = \frac{\operatorname{cov}\left(r_i ; r_j\right)^{0,T}}{\sigma_i^{0,T} \sigma_j^{0,T}}$$

and the realized 'average pairwise correlation' is equal to:

$$\rho_{averagerealised} = \frac{1}{n \times (n-1)} \sum_{i=1}^{n} \sum_{i \neq j} \rho_{i,j}^{o,T}$$

- **CLEAN CORRELATION**

Consider an index (i.e. a basket) with *n* stocks. If we replicate the index, we constitute a basket with the following volatility:

$$\sigma_I^{\,2} = \sum_{i=1}^{n} \omega_i^{\,2} \sigma_i^{\,2} + \sum_{i=1, j \neq i}^{n} \omega_i \omega_j \sigma_i \sigma_j \rho_{ij}$$

Assuming a single average value for the off-diagonal correlation, we can define the implied correlation, namely an 'average correlation' or 'clean correlation' of the portfolio, as follows:

$$\rho_{imp} = \frac{\sigma_I^{\,2} - \sum_{i=1}^{n} \omega_i^{\,2} \sigma_i^{\,2}}{\left(\sum_{i=1}^{N} \omega_i \sigma_i\right)^2 - \sum_{i=1}^{n} \omega_i^{\,2} \sigma_i^{\,2}}$$

- **DIRTY CORRELATION**

Bossu[4] assumed that, if the Index is well diversified (with more than 20 constituents), the term $\sum_{i=1}^{n} \omega_i^{\,2} \sigma_i^{\,2}$ is close to zero. Hence, a good proxy for the implied correlation, which we refer to as 'dirty correlation', is:

$$\rho = \frac{\sigma_I^{\,2}}{\left(\sum_{i=1}^{n} \omega_i \sigma_i\right)^2} = \left(\frac{IndexVolatiliy}{WeightedAverageStockVolatility}\right)^2$$

Hence, the dirty correlation can be seen as the ratio between two traded products, through variance swaps or variance dispersion trades. Moreover, this formula can be used in reverse to predict a value for index volatility given values for correlation and average single-stock volatility.

$$\sum_{i=1}^{n} \omega_i \sigma_i \times \sqrt{\rho} = \sigma_I$$

We can see that the relationship is non-linear: when correlation is low, an increase in correlation will cause greater increase in volatility than when correlation is high.

- **REALISED CORRELATION VS. IMPLIED CORRELATION**

Realised correlation can be computed using any of the above formulas with realized volatility. In the case of implied correlation, as I have briefly mentioned before we cannot derive the Implied Pairwise Correlation, but the 'clean' and 'dirty' correlation can be computed with the equations above using ATM implied volatility.

REALISED CORRELATION < IMPLIED CORRELATION

Implied correlation is the market's expectations of future realised index correlation. However, historically a noticeable trend has been for implied correlation to stand above the subsequent realized correlation, as a consequence of index volatility trading rich relative to the constituent single-stock volatility.

There are several explanations to the implied correlation risk premium:

- **Negative Correlation with market level**: Correlation tends to rise when equity markets fall, thus market participants will ask for a premium to be short correlation.

[4] Reference number 4

- **Correlation skew**: Index Skew tends to be steeper than the skew of the single stocks since market participants anticipate a higher correlation when the equity market falls, and higher correlation would imply a higher skew.
- **Need for hedge of long correlation positions**: As I have mentioned in the introduction, investment banks with significant structured products business use correlation swaps or dispersion trades to lay-off part of their correlation risk that they have built up by selling structured products.
- **Flow Activity**: Institutional investors often buy index put options to protect their portfolio and portfolio managers sell OTM calls on single stocks in order to receive the premium in exchange of capping their maximum potential upside thus resulting in a synthetic long implied correlation position. Investment banks on the other side are short correlation and too many look to buy back correlation, which again acts as a source in increasing the implied correlation.

CORRELATION SWAPS

A correlation swap is an instrument that pays the difference between the realised correlation and the strike. Normally, for the realized correlation we use the Average Pairwise Correalation which we have defined before. Thus, mathematically the payoff would be:

$$CorrSwap = N\frac{2}{n\times(n-1)}\sum_{i=1}^{n}\sum_{i\neq j}\rho_{i,j}^{o,T} - K$$

where $\rho_{i,j}^{o,T}$ is the pairwise correlation.

FAIR STRIKE

In theory, the strike of a correlation swap should trade close to the dirty implied correlation. Thus, to compute the fair strike of the correlation swap we need to replicate it using other liquid products. In fact, the correlation swap can be dynamically quasi-replicated by a variance dispersion trade, but liquidity is not enough on all markets for variance swaps, neither for every index and its components.

Moreover, even using options which are liquid enough, the implied average pairwise correlation can not be computed since it depends on the correlations between all pairs of stocks. We would need implied covariance data to calculate such implied correlation with plain vanilla options. Even if the implied covariance between pairs of single stocks could be obtained through out-performance or spread options, these instruments are neither listed nor liquid enough to provide useful estimates of covariance. Thus it is difficult to estimate the fair strike.

In reality, since the market for correlation swaps is not liquid enough, the traders determine the strike of the correlation swaps by the supply and demand of the market participants. Moreover, the traders have a rule of thumb to calculate a lambda which transfers the realised pairwise correlation to implied pairwise correlation and vice versa, which gives an intuitive range where the fair strike should lie between. The lambda is calculated from existing correlation trades. First we derive the implied pairwise correlation from the Mark to Market values.

$$MarkToMarket = \rho_{realised} \times T_{\%elapsed} + \rho_{implied} \times \left(1 - T_{\%elapsed}\right)$$

$$\Leftrightarrow \rho_{implied} = \frac{MarkToMarket - \rho_{realised} \times T_{\%elapsed}}{1 - T_{\%elapsed}}$$

And then we deduce the lambda by the following equation.

$$\rho_{implied} = \lambda + (1 - \lambda)\rho_{realised}$$

$$\Leftrightarrow \lambda = \frac{\rho_{implied} - \rho_{realised}}{1 - \rho_{realised}}$$

DISPERSION TRADES

Dispersion Trade's payoff slightly differs from the 'Dispersion' we have defined above. It is because dispersion trade have been traded using equity options in the past but since the liquidity of variance swaps increased, the variance dispersion has become the standard correlation vehicle in the Equity Derivative world. Here, we introduce two different types of dispersion trades, namely vega-neutral dispersion trade and vega-neutral dispersion trade, which differ in the weighting schemes.

VEGA-NEUTRAL DISPERSION

The 'Plain Vanilla' dispersion consists in selling index variance swaps while buying single stocks variance swaps.Thus the dispersion P&L is given by:

$$Disp = \left(\sum_{i=1}^{N} \frac{N_i}{2K_i}\left(\sigma_i^2 - K_i^2\right)\right) - \frac{N_I}{2K_I}\left(\sigma_I^2 - K_I^2\right)$$

In particular, this dispersion trade is called "Vega-Neutral" as same amount of vega notional is bought and sold thus making the trader immune against short moves in volatility.

The P&L of a vega-neutral dispersion trade can be restated as:

$$Disp = N\left(\left(\sum_{i=1}^{N} \frac{\omega_i}{2K_i}\left(\sigma_i^2 - K_i^2\right)\right) - \frac{1}{2K_I}\left(\sigma_I^2 - K_I^2\right)\right)$$

$$= \frac{N}{2}\left(\sum_{i=1}^{N} \frac{\omega_i}{K_i}\left(\sigma_i^2\right) - \frac{1}{K_I}\left(\sigma_I^2\right)\right) - \frac{N}{2}\left(\sum \omega_i K_i - K_I\right)$$

LONG DISPERSION = SHORT CORRELATION

We demonstrate how engaging in a dispersion trade gives us exposure to correlation.

As the dirty correlation is given by $\rho = \dfrac{{\sigma_I}^2}{\left(\sum_{i=1}^{n} \omega_i \sigma_i\right)^2}$,

Differentiating the P&L with respect to correlation yields a value always negative.

$$Disp = \frac{N}{2}\left(\sum_{i=1}^{N} \frac{\omega_i}{K_i}\left(\sigma_i^2\right) - \frac{\rho}{K_I}\left(\sum_{i=1}^{n} \omega_i \sigma_i\right)^2\right) - \frac{N}{2}\left(\sum \omega_i K_i - K_I\right)$$

$$\frac{\partial Disp}{\partial \rho} = -\frac{N}{2K_I}\left(\sum_{i=1}^{n} \omega_i \sigma_i\right)^2 \leq 0$$

Hence, being long dispersion trade implies being short realized correlation. Below is a table that summarizes this.

Dispersion	Correlation	Index Leg	Stock Leg
Long Dispersion	Short Correlation	Short Index Variance	Long Stocks Variance
Short Dispersion	Long Correlation	Long Index Variance	Short Stocks Variance

POSITIVE SENSITIVITY TO (AGGREGATED) SINGLE STOCK VOLATILITY

So far we have talked about only the case of a Dispersion Trade we call ‘vega-neutral’, but this is not a pure correlation trade as the volatility exposure is not null. To analyze the Dispersion Trade’s sensitivity to single stock volatility, we rewrite the Vega-neutral Dispersion trade:

$$\begin{aligned} Disp_{vega-neutral} &= \left(\sum_{i=1}^{N} \frac{N_i}{2K_i}\left(\sigma_i^2 - K_i^2\right)\right) - \frac{N_I}{2K_I}\left(\sigma_I^2 - K_I^2\right) \\ &= \left(\sum_{i=1}^{N} \frac{N_i}{2K_i}\left(\sigma_i^2 - K_i^2\right)\right) - \frac{N_I}{2K_I}\left(\rho\left(\sum_{i=1}^{N} \omega_i \sigma_i\right)^2 - K_I^2\right) \end{aligned}$$ [5]

Then the Dispersion Trade’s sensitivity to single stock volatility is given by:

$$\begin{aligned} \frac{\partial Disp}{\partial \sigma_i} &= N\left(\omega_i \frac{\sigma_i}{K_i} - \omega_i \frac{\rho}{K_I} \sum_{i=1}^{N} \omega_i \sigma_i\right) \\ &= N\left(\omega_i \frac{\sigma_i}{K_i} - \omega_i \frac{\rho}{\sqrt{\overline{\rho}}} \frac{\sum_{i=1}^{N} \omega_i \sigma_i}{\sum_{i=1}^{N} \omega_i K_i}\right) \end{aligned}$$ [6]

[5] using the dirty realised correlation equation
[6] using the dirty implied correlation equation: notation $\overline{\rho}$

$$= N\omega_i m\left(1 - \frac{\rho}{\sqrt{\overline{\rho}}}\right)$$ [7]

ω_i and m are always positive and $\frac{\rho}{\sqrt{\overline{\rho}}}$ is smaller than 1 since correlation lies between 0 and 1. Hence, we can see that while a vega-neutral dispersion trade provides a negative exposure to correlation, it is positively sensitive to single stock volatility. In particular, it can be proved that the sensitivity of a dispersion trade with respect to the 'overall' single stocks volatility exposure is positive[8]:

$$\sum_{i=1}^{N}\frac{\partial Disp}{\partial \sigma_i} = N\sum_{i=1}^{N}\omega_i\left(\frac{\sigma_i}{K_i} - \frac{\rho\sum_{i=1}^{N}\omega_i\sigma_i}{\sqrt{\overline{\rho}}\sum_{i=1}^{N}\omega_i K_i}\right)$$

$$= N\left(\sum_{i=1}^{N}\omega_i\frac{\sigma_i}{K_i} - \frac{\rho\sum_{i=1}^{N}\omega_i\sigma_i}{\sqrt{\overline{\rho}}\sum_{i=1}^{N}\omega_i K_i}\right)$$

And this value is likely to be positive since $\frac{\rho}{\sqrt{\overline{\rho}}}$ is likely to be lower than one. Hence, a vega-neutral dispersion trade is not a pure correlation trade but yields a positive sensitivity to realized volatility. Such a dispersion trade would therefore underperform in a falling volatility environment, all other things being equal.

THETA-NEUTRAL DISPERSION

There is another way to capture a more 'pure' correlation risk premium without being exposed to volatility movements. One way to reduce the exposure to single stock volatility is to increase the initial index Vega that is sold in the following proportion:

$$N_I\frac{\sum_{i=1}^{N}\omega_i K_i}{K_I} = N_I\frac{1}{\sqrt{\overline{\rho}}}$$

In this case, the dispersion is called 'theta-neutral' or 'correlation-weighted' and has a P&L equal to:

$$Disp_{theta-neutral} = \left(\sum_{i=1}^{N}\frac{N_i}{2K_i}\left(\sigma_i^2 - K_i^2\right)\right) - \frac{N_I}{2K_I\sqrt{\overline{\rho}}}\left(\sigma_I^2 - K_I^2\right)$$

$$= \frac{N_I}{2}\left(\sum_{i=1}^{N}\frac{\omega_i}{K_i}\left(\sigma_i^2\right) - \frac{1}{K_I\sqrt{\overline{\rho}}}\left(\sigma_I^2\right)\right) - \frac{N_I}{2}\left(\sum_{i=1}^{N}\omega_i K_i - \frac{K_I}{\sqrt{\overline{\rho}}}\right)$$

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[7] For simplification reasons we assume that traders over or undervalue single stocks implied volatility but by the same amount of bias for all stocks. Thus: $\sigma_i = mK_i$

[8] Reference number 8

$$= \frac{N_I}{2}\left(\sum_{i=1}^{N}\frac{\omega_i}{K_i}\left(\sigma_i^2\right) - \frac{1}{K_I\sqrt{\overline{\rho}}}\left(\sigma_I^2\right)\right)$$ [9]

$$= \frac{N_I}{2}\left(\sum_{i=1}^{N}\frac{\omega_i}{K_i}\left(\sigma_i^2\right) - \frac{\rho}{K_I\sqrt{\overline{\rho}}}\left(\sum_{i=1}^{N}\omega_i\sigma_i\right)^2\right)$$

We can see that the implied part of the dispersion trade disappears, which can be explained by the fact the higher index Vega notional compared to offset the initial strike price different between single stocks and index. If single stocks and index variances move by the same absolute amount, the realised dispersion for that day will be equal to zero which is why this trade is called 'theta-neutral'. In other words, the single stocks and the index legs of the trade have the same time decay.

THEORETICAL VALUE

Moreover, we can derive the theoretical value of a vega-neutral dispersion trade. Below is the demonstration of the calculation:

$$Disp = \left(\sum_{i=1}^{N}\frac{N_i}{2K_i}\left(\sigma_i^2 - K_i^2\right)\right) - \frac{N_I}{2K_I}\left(\sigma_I^2 - K_I^2\right)$$

$$= \frac{1}{\sum_k \omega_k K_k}\sum_{i=1}^{N}\frac{N_i}{2}\left(\sigma_i^2 - K_i^2\right) - \frac{N_I}{2K_I}\left(\sigma_I^2 - K_I^2\right)$$ [10]

$$= \frac{N_I K_I}{\left(\sum_i \omega_i K_i\right)^2}\sum_{i=1}^{N}\frac{\omega_i}{2}\left(\sigma_i^2 - K_i^2\right) - \frac{N_I}{2K_I}\left(\sigma_I^2 - K_I^2\right)$$ [11]

$$= \frac{N_I}{2K_I}\left(\frac{{K_I}^2}{\left(\sum_i \omega_i K_i\right)^2}\sum_{i=1}^{N}\omega_i\left(\sigma_i^2 - K_i^2\right) - \left(\sigma_I^2 - K_I^2\right)\right)$$

$$= \frac{N_I}{2K_I}\left(\rho_K\sum_{i=1}^{N}\omega_i\left(\sigma_i^2 - K_i^2\right) - \left(\sigma_I^2 - K_I^2\right)\right)$$

$$= \frac{N_I}{2K_I}\left(\rho_K\left(\left(\sum_i \omega_i\sigma_i\right)^2 - \left(\sum_i \omega_i K_i\right)^2\right) - \left(\sigma_I^2 - K_I^2\right)\right)$$ [12]

[9] $\sum_{i=1}^{N}\omega_i K_i - \frac{K_I}{\sqrt{\overline{\rho}}} = 0$ by definition of implied dirty correlation ($\overline{\rho}$)

[10] In case the standard deviation of the singles is low, which should be the case for a dispersion trade with more than 20 constituents, we have: $\frac{1}{K_i} \approx \frac{1}{\sum_i \omega_i K_i}$

[11] Here we consider a Correlation-weighted dispersion (theta-neutral Dispersion) which the total Vega notional of the singles is multiplied by the square root of the index correlation;

$$= \frac{N_I}{2K_I}\left(\rho_K\left(\frac{\sigma^2}{\rho} - \frac{K^2}{\rho_K}\right) - \left(\sigma_I^2 - K_I^2\right)\right)$$ [13]

$$= \frac{N_I}{2K_I}\left(\frac{\sigma^2}{\rho}\sigma^2 - \sigma^2\right)$$

$$= \frac{N_I}{2K_I}\left(\frac{\sigma^2}{\rho}\left(\rho_K - \rho\right)\right)$$

$$= \frac{N_I}{2K_I}\left(\sum_i \omega_i \sigma_i\right)^2 \left(\rho_K - \rho\right)$$

$$= \frac{N_I}{2K_I}\sum_i \omega_i {\sigma_i}^2 \left(\rho_K - \rho\right)$$ [14]

In conclusion, we have:

$$\boxed{Disp = \frac{N_I}{2K_I}\sum_i \omega_i {\sigma_i}^2 \left(\rho_K - \rho\right)} \quad (*)$$

Which indicates the payoff of the dispersion is the spread between implied and realised correlation multiplied by the average variance of the components.

GREATER SENSITIVITY TO CORRELATION

To compare, the theta-neutral dispersion ‘s sensitivity to dirty correlation is higher than in the case of a vega neutral dispersion which is mainly due to the fact that more index variance swap is sold:

$$\frac{\partial Disp_{theta-neutral}}{\partial \rho} = -\frac{N}{2\sqrt{\rho}K_I}\left(\sum_{i=1}^{n}\omega_i\sigma_i\right)^2 \le \frac{\partial Disp}{\partial \rho} = -\frac{N}{2K_I}\left(\sum_{i=1}^{n}\omega_i\sigma_i\right)^2 \le 0$$

LESS SENSITIVE TO (AGGREGATED) SINGLE STOCK VOLATILITY

And if we look at the sensitivity to single stock volatility:

---

[12] $N_i = \omega_i \times N_I \times \sqrt{\rho_K} = \omega_i \times N_I \times \frac{K_I}{\sum_{i=1}^{n}\omega_i K_i}$

[13] $\sum_i \omega_i {\sigma_i}^2 \approx \left(\sum_i \omega_i \sigma_i\right)^2$

[14] As of the footnote <16>

$$\frac{\partial Disp_{theta-neutral}}{\partial \sigma_i} = N_I \omega_i \left(\frac{\sigma_i}{K_i} - \frac{\rho}{\sqrt{\bar{\rho}}} \frac{\sum_{i=1}^{N} \omega_i \sigma_i}{\sum_{i=1}^{N} \omega_i K_i} \right)$$

$$= N_I m \left(1 - \frac{\rho}{\bar{\rho}} \right)^{15}$$

Compared to the vega-neutral dispersion trade, the sensitivity is closer to zero as $\frac{\rho}{\bar{\rho}}$ is closer to one than $\frac{\rho}{\sqrt{\bar{\rho}}}$.

The result holds when we aggregate all the single stock volatility as it yields:

$$\sum_{i=1}^{N} \frac{\partial Disp}{\partial \sigma_i} = N \left(\sum_{i=1}^{N} \omega_i \frac{\sigma_i}{K_i} - \frac{\rho \sum_{i=1}^{N} \omega_i \sigma_i}{\bar{\rho} \sum_{i=1}^{N} \omega_i K_i} \right)$$

The following table summarizes the comparison between the vega-neutral dispersion and the theta-neutral dispersion:

Dispersion Type	Correlation Sensitivity	Single Stock Sensitivity	Aggregated Single Stock Sensitivity
Vega-neutral	$-\frac{N}{2K_I}\left(\sum_{i=1}^{n} \omega_i \sigma_i\right)^2$	$N\omega_i m\left(1 - \frac{\rho}{\sqrt{\bar{\rho}}}\right)$	$N\left(\sum_{i=1}^{N} \omega_i \frac{\sigma_i}{K_i} - \frac{\rho \sum_{i=1}^{N} \omega_i \sigma_i}{\sqrt{\bar{\rho}} \sum_{i=1}^{N} \omega_i K_i}\right)$
vs.	▼	▼	▼
Theta-neutral (Correlation-weighted)	$-\frac{N}{2\sqrt{\bar{\rho}} K_I}\left(\sum_{i=1}^{n} \omega_i \sigma_i\right)^2$	$N_I m\left(1 - \frac{\rho}{\bar{\rho}}\right)$	$N\left(\sum_{i=1}^{N} \omega_i \frac{\sigma_i}{K_i} - \frac{\rho \sum_{i=1}^{N} \omega_i \sigma_i}{\bar{\rho} \sum_{i=1}^{N} \omega_i K_i}\right)$

In short, the theta-neutral strategy is more sensitive to correlation and less sensitive to volatility compared to a vega-neutral strategy. Thus, vega-neutral trade would be an efficient way to own volatility, whereas correlation-weighted dispersion trade would profit more in the moves of correlation.

PROFITABLE STRATEGY, 'SHORT DISPERSION'

In particular, for the same reasons implied correlation tends to be above the realised correlation, historically the sale of correlation through a dispersion trade has been a profitable strategy. Among some of the reasons, one is that there is the demand for index protection. Institutions usually buy protection through index volatility, hence keeping the volatility level of the index higher relative to the average volatility level of the basket. In contrast there are more sellers of single stocks volatility than there are on the index side. Moreover, since the index

[15] We use the same undervalue/overvalue assumption as the Vega-neutral dispersion trade.

is more liquid than the single stocks, it's easy to observe the extra volatility on the index. Underestimation in event risk such as bankruptcies, corporate scandals and mergers/takeover count as another reason the level of volatility of baskets is lower.

ρ_{imp} (DISPERSION TRADE) > ρ_{imp} (CORRELATION SWAP)

It has empirically been showed that the implied correlation in a dispersion trade is not equal to the strike of a correlation swap with the same maturity[16]. Indeed, the implied correlation tends to be around 10 points higher.

Part of this correlation spread of the Dispersion Trade over a Correlation Swap may be viewed as the risk premium due to the dependence of the Dispersion Trade level to the level of volatility compared with the Index Correlation level. The result (*) clearly shows that the volatility affects the notional, which is to say, the P&L. If the average volatility goes up, the notional increases. Thus, the person taking a position in a Dispersion trade rather than a Correlation Swap bears more risk due to the effect from the volatility.

Thus, just as the convexity-adjustment justifies the higher value of the Variance Swap rate compared to the Volatility Swap rate, the Dispersion Trade 'Correlation' level is higher than the weighted-average pairwise correlation measure as traded through a Correlation Swap. We further demonstrate this spread in the following section.

The effect between the correlation and the average volatility is not straightforward, although we can see a strong positive relationship from the historical data. The relationship can be considered as correlation tending to increase with the average volatility (high volatility regime = high correlation regime). Thus, when the investor is short correlation (Long Dispersion) and correlation picks up, loss are going to be higher in a similar manner to short variance.

ANALYTICAL FORMULA FOR THE SPREAD

Let $\sigma_I^2 = \sum_{i=1}^{n} \omega_i^2 \sigma_i^2 + \sum_{i=1, j\neq i}^{n} \omega_i \omega_j \sigma_i \sigma_j \rho_{ij}$ implied volatility, and $\hat{\sigma}_I^2 = \sum_{i=1}^{n} \omega_i^2 \hat{\sigma}_i^2 + \sum_{i=1, j\neq i}^{n} \omega_i \omega_j \hat{\sigma}_i \hat{\sigma}_j \hat{\rho}_{ij}$ realized volatility. By subtracting these two equalities, we get:

$$\hat{\sigma}_I^2 - \sigma_I^2 = \sum_{i=1}^{n} \omega_i^2 \left(\hat{\sigma}_i^2 - \sigma_i^2\right) + \sum_{i=1, j\neq i}^{n} \omega_i \omega_j \left(\hat{\sigma}_i \hat{\sigma}_j \hat{\rho}_{ij} - \sigma_i \sigma_j \rho_{ij}\right)$$

$$= \sum_{i=1}^{n} \omega_i^2 \left(\hat{\sigma}_i^2 - \sigma_i^2\right) + \sum_{i=1, j\neq i}^{n} \omega_i \omega_j \left[\sigma_i \sigma_j \left(\hat{\rho} - \rho\right) + \left(\hat{\sigma}_i \hat{\sigma}_j - \sigma_i \sigma_j\right)\hat{\rho}\right]$$

[16] Reference number 6 and 7

$$= \sum_{i=1}^{n} \omega_i^{\,2} \left(\hat{\sigma}_i^{\,2} - \sigma_i^{\,2} \right) + \sum_{i=1,\, j \neq i}^{n} \omega_i \omega_j \sigma_i \sigma_j \left(\hat{\rho} - \rho \right) + \sum_{i=1,\, j \neq i}^{n} \hat{\rho} \left(\hat{\sigma}_i \hat{\sigma}_j - \sigma_i \sigma_j \right)$$

Hence, we obtain the spread on the right side:

$$\left[\sum_{i=1}^{n} \omega_i^{\,2} \left(\hat{\sigma}_i^{\,2} - \sigma_i^{\,2} \right) - \left(\hat{\sigma}_I^{\,2} - \sigma_I^{\,2} \right) \right] - \sum_{i=1,\, j \neq i}^{n} \omega_i \omega_j \sigma_i \sigma_j \left(\hat{\rho} - \rho \right) = \sum_{i=1,\, j \neq i}^{n} \hat{\rho} \left(\hat{\sigma}_i \hat{\sigma}_j - \sigma_i \sigma_j \right)$$

This is equivalent to taking a short position in a dispersion trade through variance swaps (short the components' variance swaps and long the index variance swap) and a long position in a Correlation Swap.

STRATEGY

As mentioned above, selling correlation through dispersion has historically been profitable. Thus, we will take a long position on a dispersion trade and hedge our position by taking a long position on a Correlation Swap. In case of a volatility spike, the loss on the dispersion trade will be compensated by the gain in the correlation swap. In short the strategy we will take is:

Long Dispersion Trade
Long Correlation Swap

In particular, based on an existing Correlation Swap deal in Lyxor Asset Management, we build a dispersion trade with their implied volatilities at the date of inception as their strikes. We take a Correlation Swap whose underlyings are the components of the Eurostoxx50 with a maturity of 9 months and the strike of 52%. Thus:

Strike Date	13-Mar-08
Maturity date	19-Dec-08
Strike =K	52
Nominal = N	10 000 000
Number of Stocks = n	50

Weight = ω_i	**Stocks and Indice (Ticker)**	**Vol Strike =** K_i	**Variance Cap**
0.85%	ACA FP	39.29	96.47
0.86%	AGN NA	42.46	112.68
0.97%	AI FP	28.93	52.31
0.64%	ALU FP	50.29	158.06
2.89%	ALV GR	35.60	79.19
2.04%	BAS GR	28.24	49.85
1.89%	BAY GR	33.55	70.35
2.66%	BBVA SM	31.41	61.67
1.20%	BN FP	29.73	55.23
2.77%	BNP FP	37.51	87.95
1.11%	CA FP	29.55	54.57
2.29%	CS FP	38.93	94.72
3.21%	DAI GR	34.51	74.44
0.93%	DB1 GR	42.22	111.41
1.86%	DBK GR	34.63	74.94
1.09%	DG FP	36.55	83.51
1.81%	DTE GR	28.75	51.64
1.46%	ENEL IM	23.96	35.89
2.62%	ENI IM	26.17	42.80
3.74%	EOA GR	27.42	46.99
2.11%	FORA NA	40.31	101.55
5.31%	FP FP	27.40	46.92
2.06%	FTE FP	29.08	52.86

1.68%	G IM	25.16	39.57
2.21%	GLE FP	42.20	111.31
1.86%	IBE SM	30.87	59.56
2.84%	INGA NA	39.94	99.72
2.04%	ISP IM	27.62	47.67
0.97%	MC FP	32.26	65.03
1.89%	MTP FP	42.67	113.77
1.24%	MUV2 GR	29.06	52.79
4.62%	NOK1V FH	42.35	112.12
0.98%	OR FP	30.64	58.69
1.31%	PHIA NA	31.25	61.02
0.90%	REP SM	33.04	68.23
0.94%	RNO FP	42.72	114.04
1.90%	RWE GR	26.19	42.88
2.65%	SAN FP	29.70	55.14
3.82%	SAN SM	31.56	62.24
1.44%	SAP GR	30.36	57.61
1.18%	SGO FP	37.01	85.62
3.33%	SIE GR	35.44	78.48
1.00%	SU FP	37.81	89.33
2.24%	SZE FP	30.45	57.96
4.06%	TEF SM	28.87	52.11
1.02%	TIT IM	33.03	68.19
3.18%	UCG IM	36.45	83.03
1.65%	UNA NA	27.33	46.69
1.54%	VIV FP	30.24	57.15
1.09%	VOW GR	21.91	30.00
	SX5E	25.54	40.77

The payoff of our strategy will be:

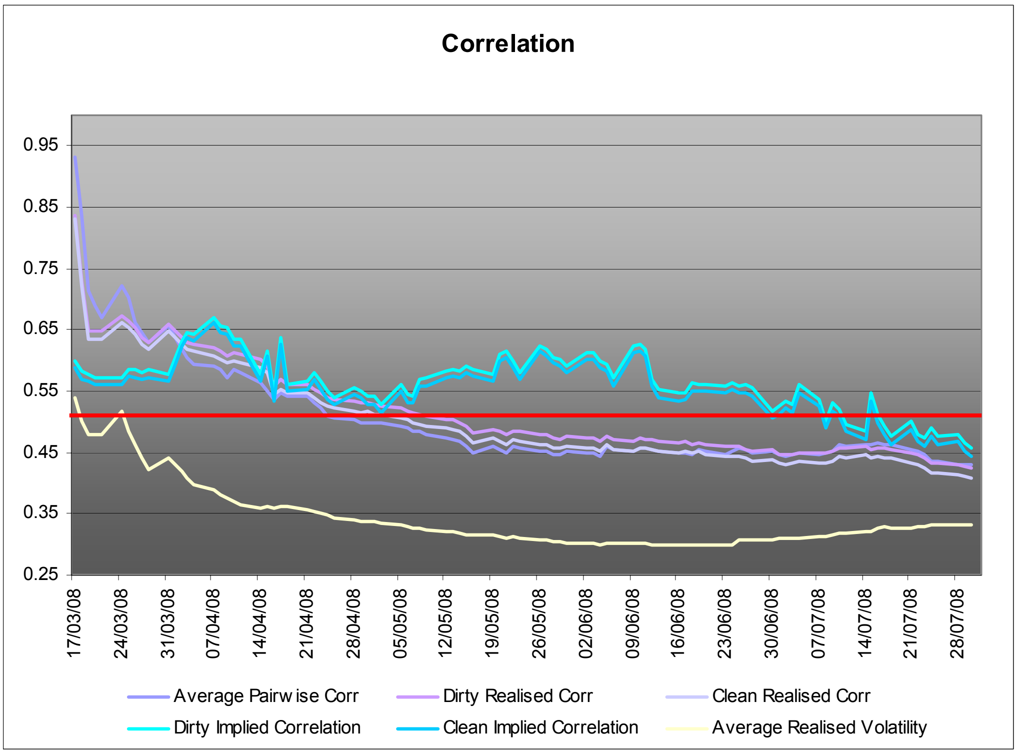
$$Strategy = N_I \times \frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{i \neq j} \frac{\sum_{i=0}^{T} \left(r_{i,t} - \overline{r_i}\right)\left(r_{j,t} - \overline{r_j}\right)}{\sqrt{\sum_{i=0}^{T} \left(r_{i,t} - \overline{r_i}\right)^2} \sqrt{\sum_{i=0}^{T} \left(r_{j,t} - \overline{r_j}\right)^2}} - K + \left(\sum_{i=1}^{n} \frac{N_i}{2K_i} \left(\sigma_i^2 - K_i^2\right) \right) - \frac{N_I}{2K_I} \left(\sigma_I^2 - K_I^2\right)$$

Using the estimation for the value of the Dispersion Trade we have:

$$\approx N_I \times \frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{i \neq j} \frac{\sum_{i=0}^{T} \left(r_{i,t} - \overline{r_i}\right)\left(r_{j,t} - \overline{r_j}\right)}{\sqrt{\sum_{i=0}^{T} \left(r_{i,t} - \overline{r_i}\right)^2} \sqrt{\sum_{i=0}^{T} \left(r_{j,t} - \overline{r_j}\right)^2}} - K + \frac{N_I}{2K_I} \left(\sum_i \omega_i \sigma_i \right)^2 \left(\rho_K - \rho\right)$$

I have calculated the correlations and the P&L of the strategy in excel using VBA. I have included the workbook in <Appendix A> and the codes in <Appendix B>. The following are the results.

CORRELATION



- **Different ways to calculate correlations**

  I have presented three ways to calculate correlation above. For the realised correlation, we can see that the clean realised correlation and dirty realised correlation has an error, even thought smaller than 0.01, with respect to the average pairwise correlation which is the most accurate method (the clean correlation assumes equal correlation for all pairs). And we can see that the dirty correlation which is again a shortcut of the clean correlation overestimates the correlation but not by much. Actually, the distance between the dirty and clean measures do not exceed a few correlation points, but the distance between the clean and average Pairwise measures can occasionally be significant (more than 10 correlation points)[17]. In fact, it has been shown that in short-term the two measures are almost identical, but as it gets to long-term the change in stock weightings make the error.[18] This is observable throughout the period and right after inception we can see the significant distance between clean and average pairwise correlation. In the case of implied correlation, first, it is not possible to calculate pairwise correlation for the implied correlation, since it is difficult to estimate the implied pairwise covariance. Secondly, we can see again that the dirty correlation overestimates the implied correlation as it was the case for realised correlation.

- **Implied Correlation tends to stand above Realised Correlation**

[17] Reference Number 9
[18] Reference Number 11

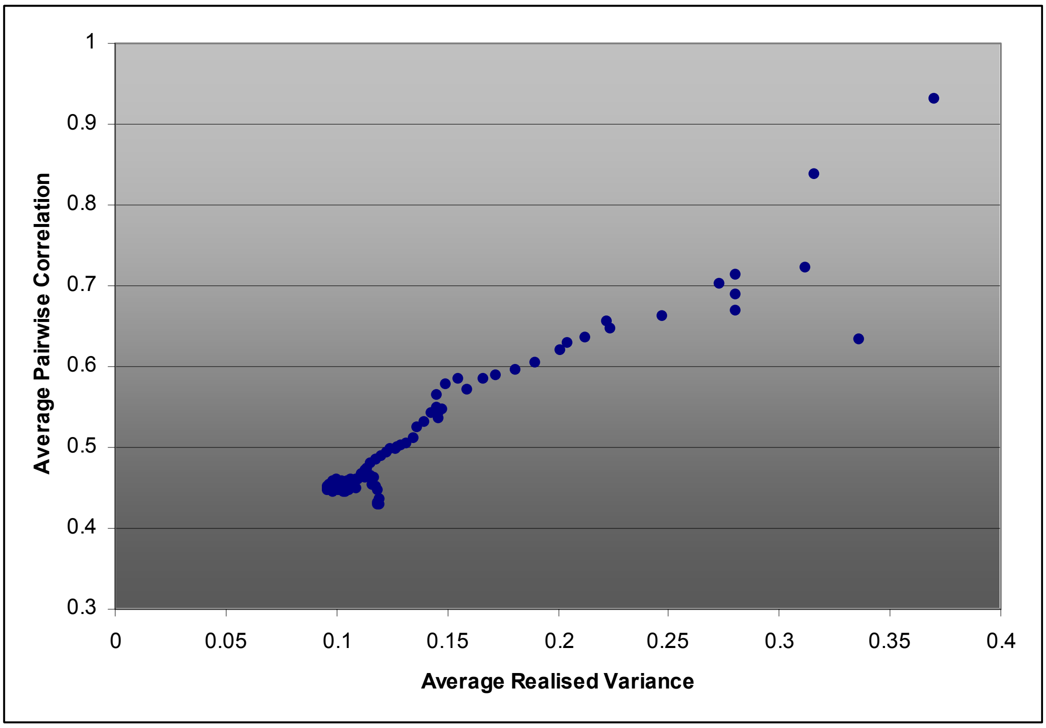
We have seen before that implied correlation tends to be above realised correlation. Even though there are some periods that they show an opposite behavior, we can conclude that this is the dominant case.

- **Implied Correlation of the Dispersion Trade tends to stay above the Strike of the Correlation Swap**

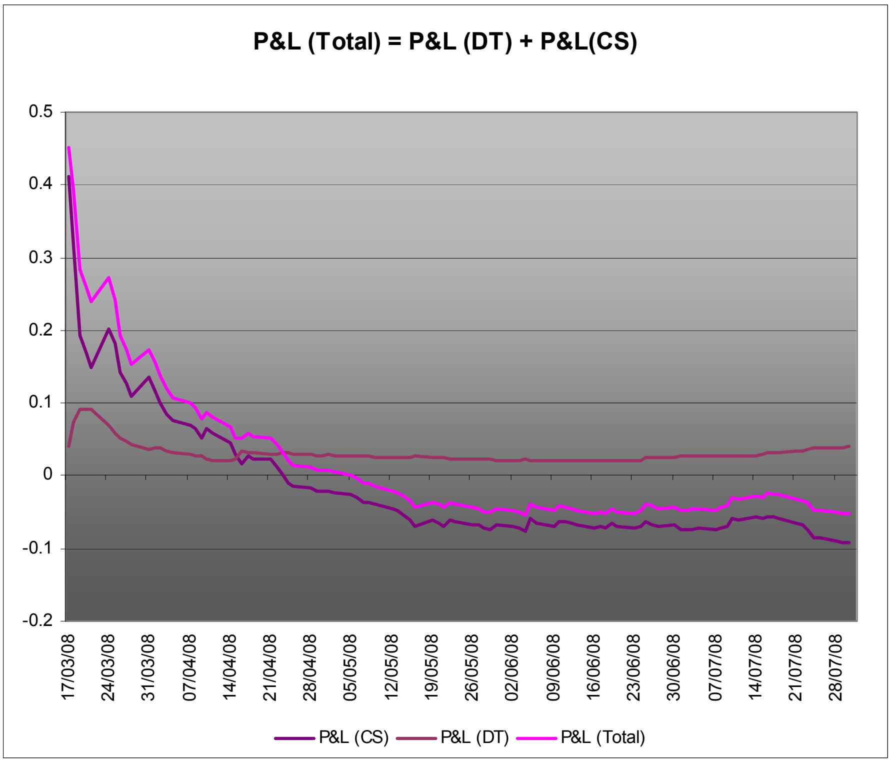
  Also, as we have mentioned above due to the risk premium for baring additional risk of the level of volatility, the implied correlation of Dispersion Trade tends to be higher than the strike of a correlation swap. We can see that except for the last few periods the implied volatility (which is marked in light blue) is above the strike of the correlation swap (which is marked in a straight red line).*

- **Average Realised Volatility of the components of the Indice and the Realised Correlation of the Indice have the same trend**

  The last point we have seen before is that the when the volatility rises the correlation tends to rise also and vice versa. We can see here that they are drawing an almost parallel line on the graph.



PROFIT & LOSS



- **P&L of the Correlation Swap is decreasing**

  From the graph above we can see that the realised pairwise correlation is continuously decreasing pass the level of the strike. As a result, the P&L continues to drop.

- **P&L of the Dispersion Trade is positive**

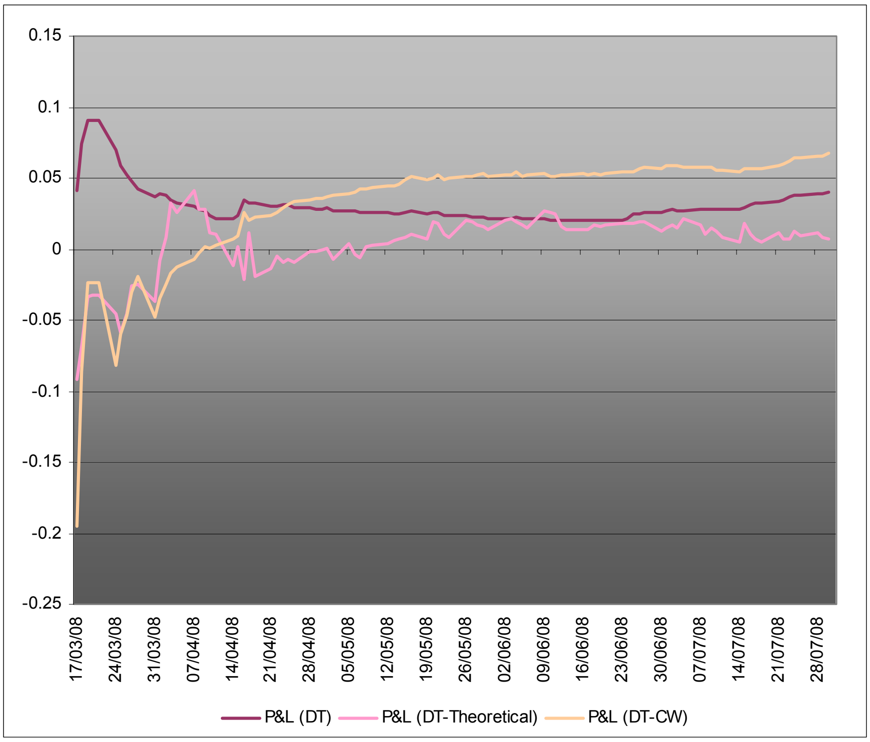
  We can see that even if a dispersion trade is also a method to invest in correlation that it is maintaining a positive P&L, as we have said before that historically this is the case.

- **Hedging effect**

  Our aim when we constructed a long dispersion trade and a long correlation swap strategy was in belief that the correlation swap would act as a hedge to the dispersion swap. However, we can see that the correlation swap is making the P&L more drastic than flattening it.

VEGA-NEUTRAL VS. THETA-NEUTRAL

In the graph below we can compare the different dispersion trades. The first, marked in maroon, is the vega-neutral dispersion. The third, marked in peach, is the theta-weighted dispersion (from now on we will call it correlation-weighted) where we multiply the nominal of the index by the inverse of the implied correlation at inception. The second series, in pink, is a theoretical estimation of the correlation-weighted dispersion.

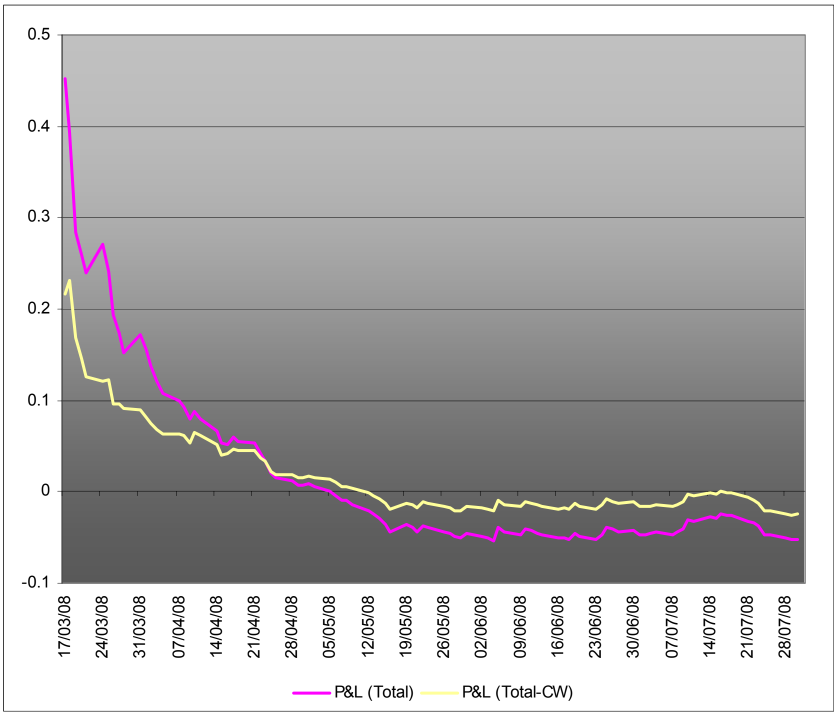


<Dispersion Trade : Vega-Neutral vs. Correlation-weighted vs. estimation of Correlation-Weighted>

- **The theoretical P&L of the Dispersion Trade has error with respect to the realised P&L**
  We have calculated before that we can estimate the value of a correlation-weighted Dispersion Trade as a spread of the implied correlation and the realised correlation times the weighted average of the variance of the components. We can see that this is only a “rough” estimation of the exact P&L.

- **Under the decreasing correlation environment, the correlation-weighted strategy shows a bigger increase in the payoff than the Vega-neutral strategy (and a bigger decrease in payoff when the correlation is high).** Looking at the Correlation and average volatility again, we can see that since volatility is always positive it is the difference in realised correlation and implied correlation who decides if the trade has a profit or a loss, and the average realised variation acts as an accelerator or a de-accelerator. Right after inception, the realised correlation is higher than the implied correlation thus showing a loss in the strategy. After is becomes the other way around and thus turns into a profit position but the spread decreases as time passes by and the average realised volatility decreases also thus accelerating the decrease in profit.

HEDGING EFFECT



Has either of the vega-neutral or correlation-weighted dispersion been well hedged through a correlation swap? Neither. Then what is the optimal weight of correlation swap and dispersion trade so that they hedge each other well? We will show this by performing a back-test, in the next section.

BACK TEST

A back-test is to test a strategy under historical market conditions, to see how well it would have performed in the past. We cannot conclude that the strategy that worked well will work as well in the future or vice versa, but a back-test can give us a good idea of the performance of the strategy under various market conditions. Thus, we conduct a back test with the same strategy before with correlation swap and dispersion trade for the maturity of 9 months and repeat the strategy over from October 2002 until July 2008.

ASSUMPTIONS

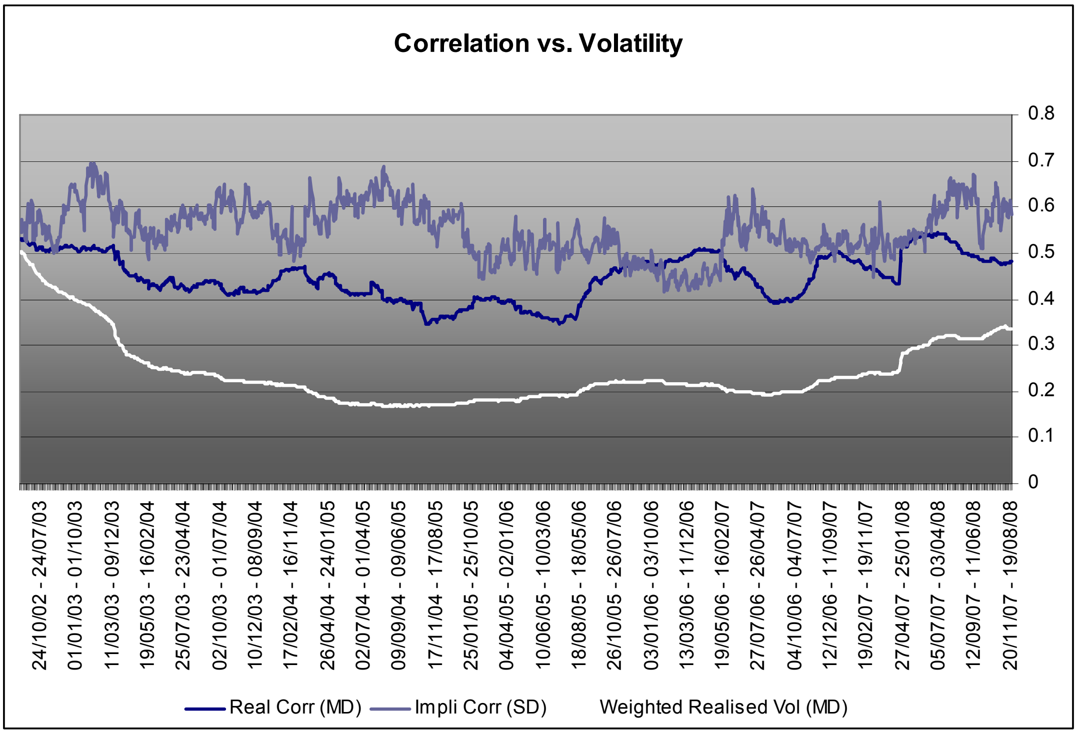
To simplify the analysis, I have posed some assumptions.

1. The components of the Eurostoxx is consistent during the analysis: In reality, the components of the Index changes when there is a corporate action

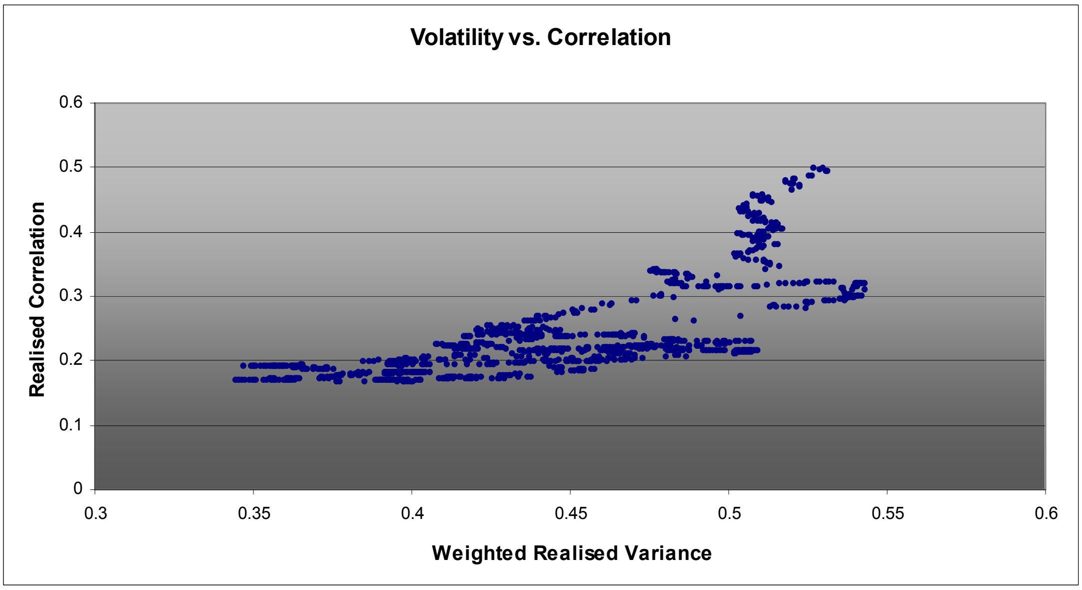
2. The weight of the components in the Eurostoxx didn't change: In reality, index weights vary over time because of changes in market capitalization. Thus, in theory, investors would need to dynamically re-adjust their single stocks variance swap positions.

VOLATILITY VS. CORRELATION

We can see that the weighted realised variance and correlation have a positive relationship. As I have explained before, variance and correlation tend to increase when the market conditions are not good.

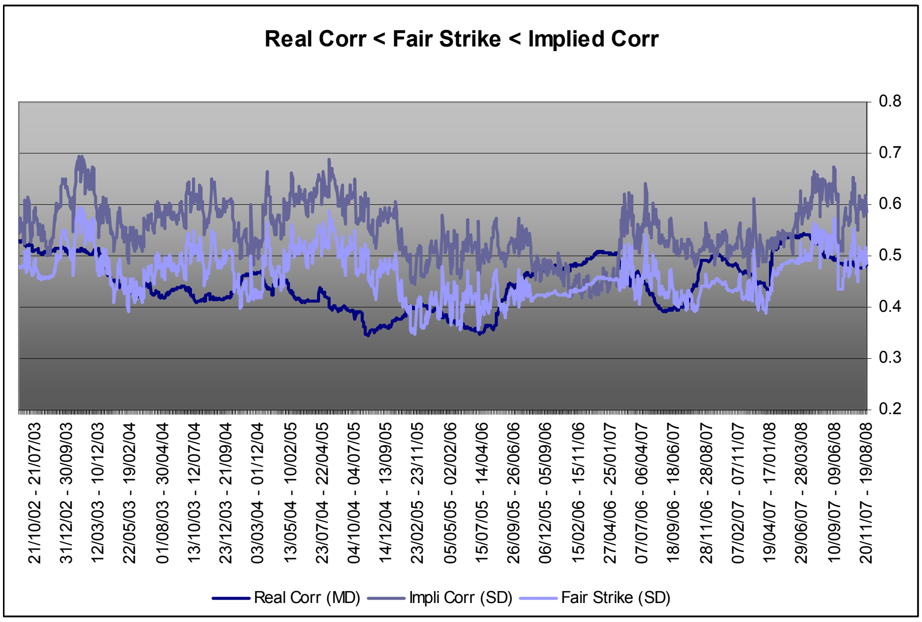


As we can see from the relationship driven from dirty correlation, $\sum_{i=1}^{n} \omega_i \sigma_i \times \sqrt{\rho} = \sigma_I$, they tend to have a non-linear relation. It is clearly demonstrated in the graph below.

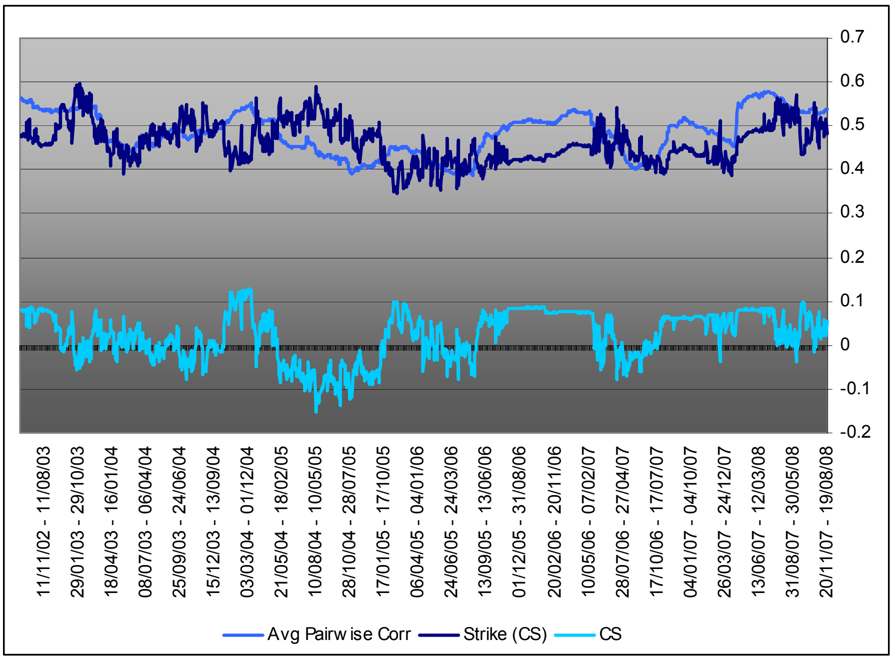


FAIR STRIKE OF CORRELATION SWAP

Since it is difficult to deduce the fair strike of the correlation swap, in accordance with the first paper in the reference, I apply the fact that it is historically 10 points below the implied correlation of the dispersion trade. Moreover, intuitively the fair strike should lie between the historical correlation of the past period and the implied correlation at the time of valuation. We can see that not all the time this is true, since until now the strike has depended more on the supply and demand for the traders. Thus, for the period the realised correlation is above the implied correlation, I take the strike as 5 points below the realised level. The following is the graph to demonstrate this. Thus, **Fair strike = Max {Implied correlation – 0.1, Realised correlation – 0.05}**

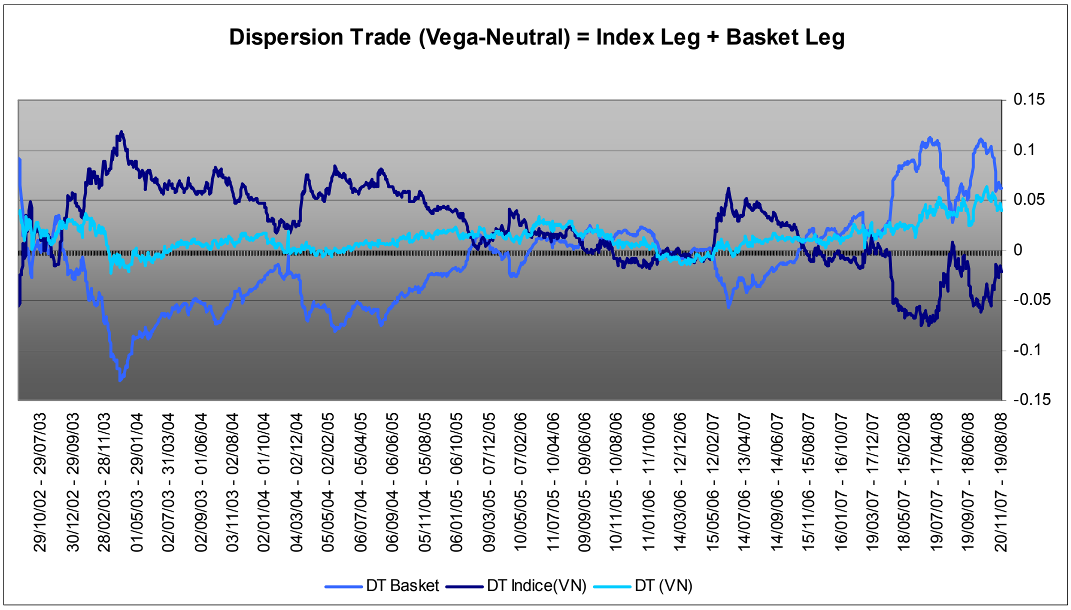


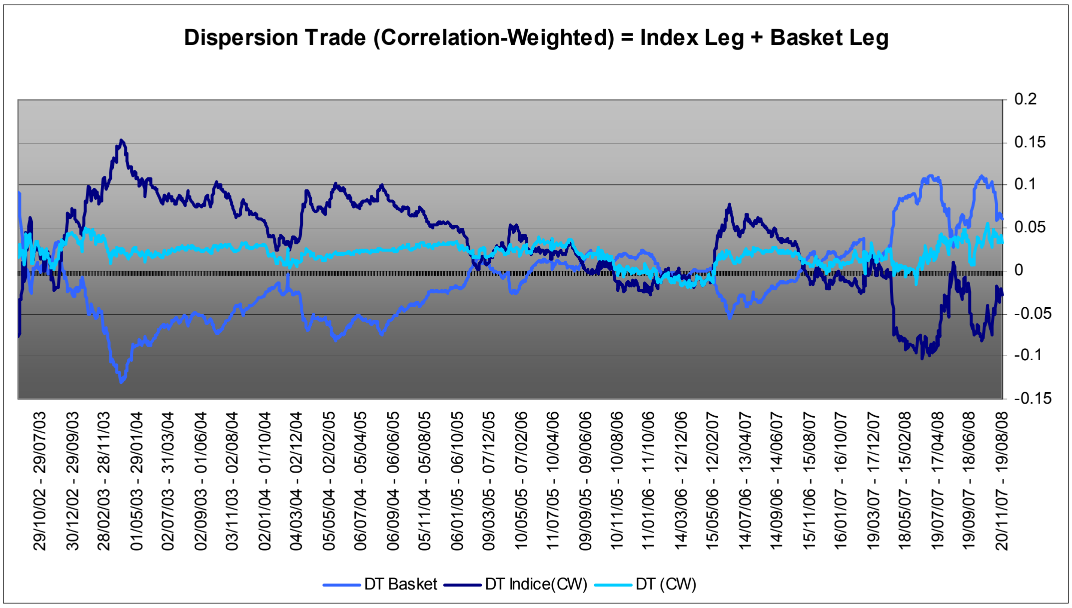
PAYOFF OF CORRELATION SWAP



PAYOFF OF DISPERSION TRADE

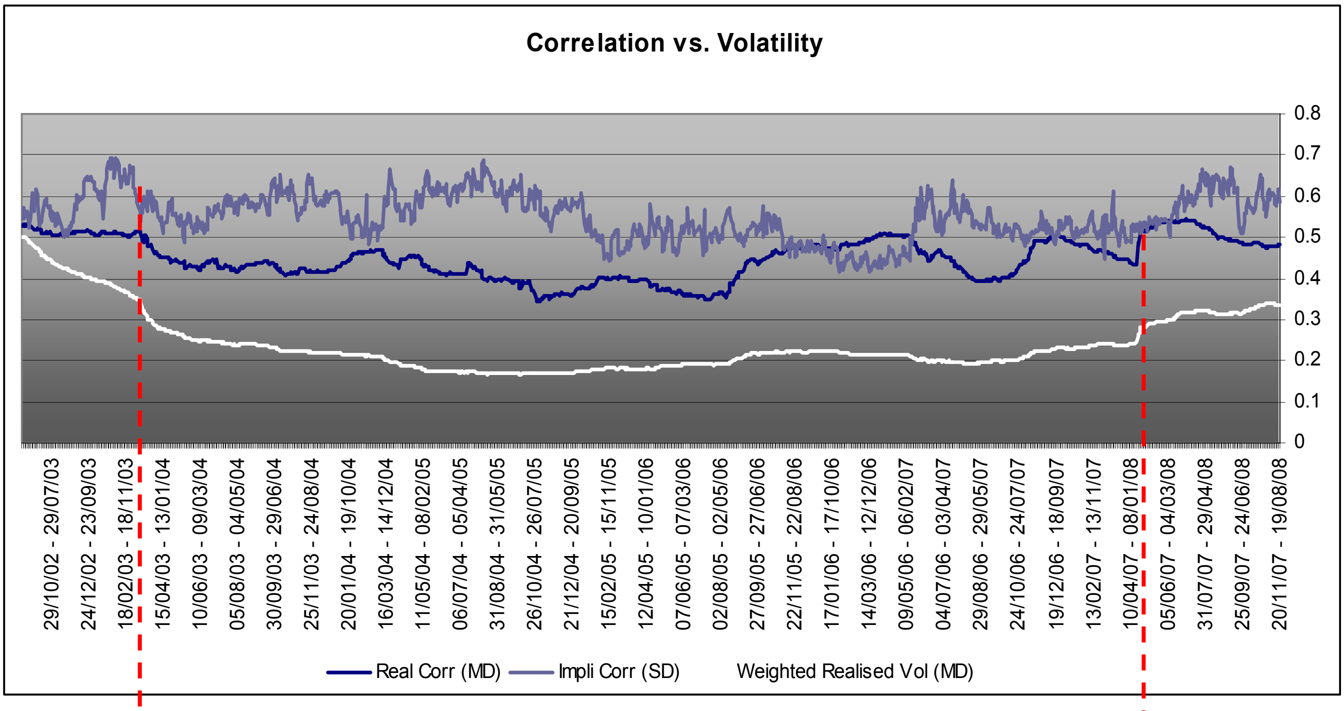
We can see that on both vega-neutral and correlation-weighted index variance is indeed trading rich relative to the variance of the components. Therefore the when the index variance has a positive return, it is greater than the loss on the components, and when it has a negative return , it is smaller than the gain on the components, thus leaving us with a positive carry globally. From the period between the maturities December 2006 and February 2007, we see that both the loss and gain on both legs are very small thus leaves us with a loss in the sum of both legs, but the loss is smaller than 1% of the vega notional. Thus, we can say that Dispersion trade is the “profit driver” in our strategy

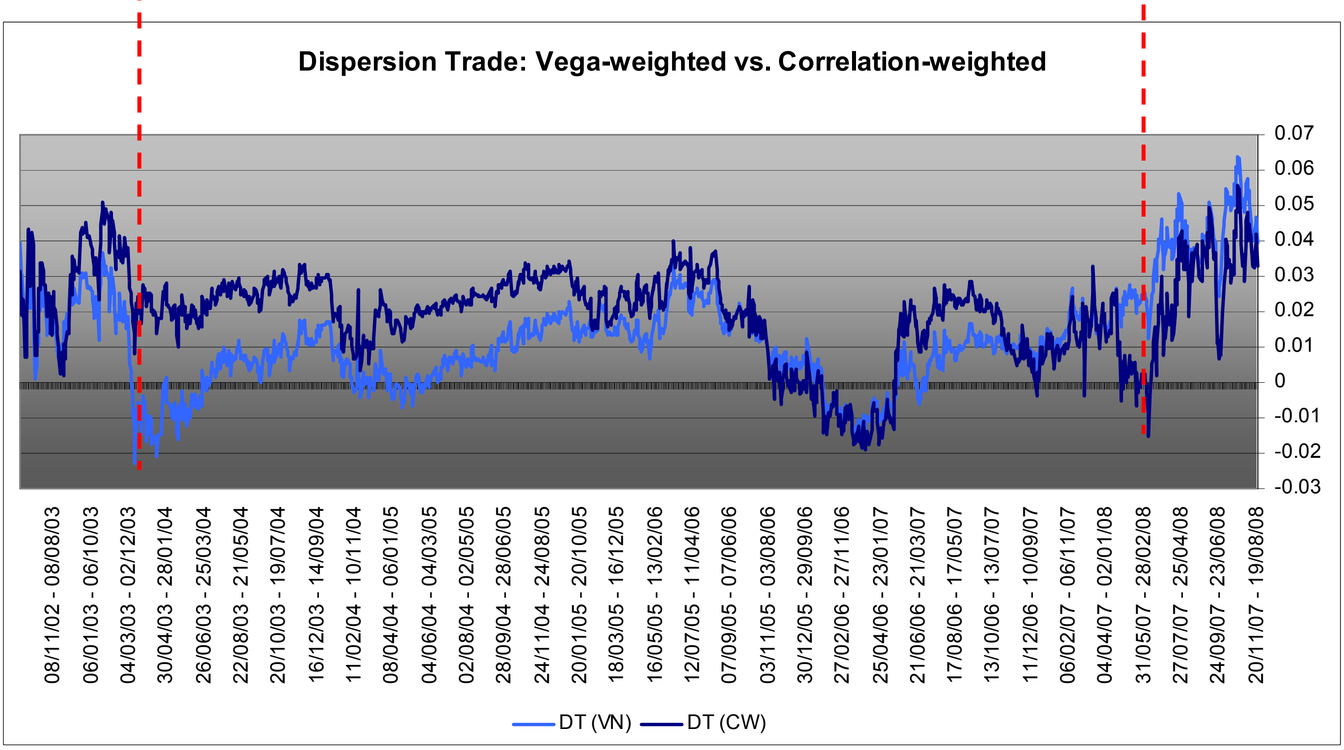




DISPERSION TRADE: VEGA-NEUTRAL VS. CORRELATION-WEIGHTED

Here, we compare the vega-neutral dispersion trade with correlation-weighted trade. We can see that indeed the correlation weighted strategy is more sensitive to the level of realised correlation. On the other hand, we see in the beginning and the end of the analysis that given the similar level of correlation, the vega- neutral strategy shows more sensitivity towards the level of weighted variance.



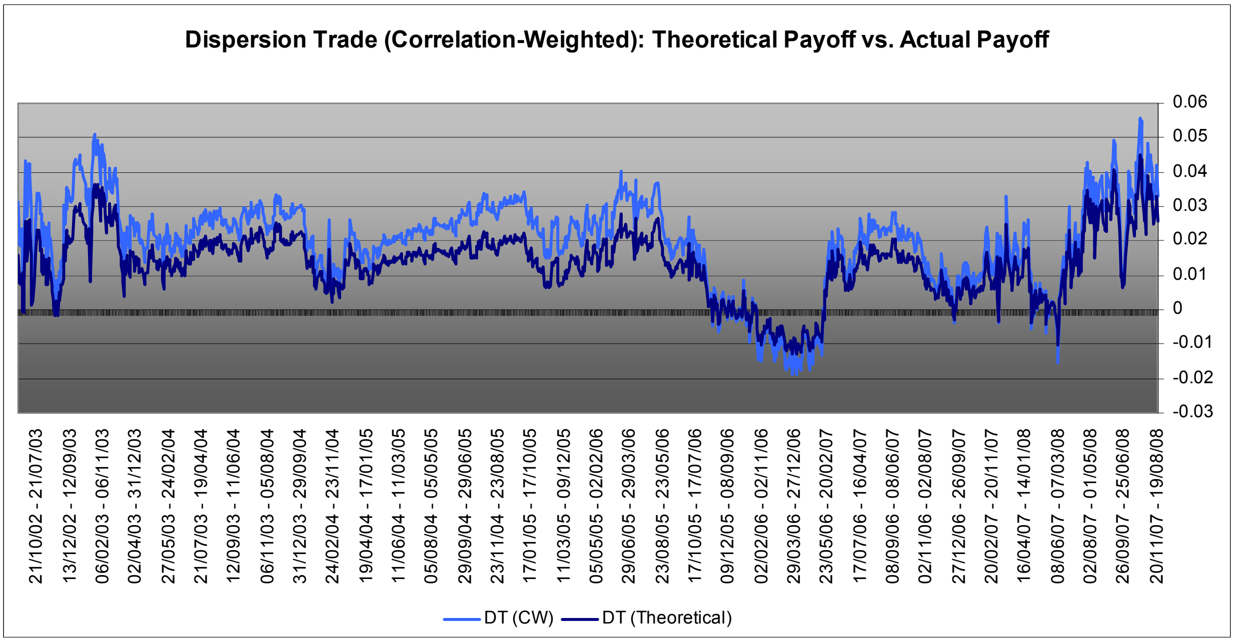


CORRELATION-WEIGHTED DISPERSION TRADE: THEORETICAL PAYOFF

We can also compare the actual payoff with the theoretical payoff we have computed before.

$$\sum_i \omega_i \sigma_i^{\,2}(\rho_K - \rho)$$

We see that it has some errors, but the average error over the 6 years where I performed the back test was less than 0.6%.



HEDGING STRATEGY

So I am trying to find an optimal weight of correlation swap and dispersion trade that will enable the correlation swap to act as a hedge to the dispersion trade i.e. it will protect the dispersion trade in case of a correlation spike. In particular, here we take the correlation-weighted dispersion trade to have maximum exposure on correlation and minimum exposure on the level of volatility. And we suppose that the theoretical payoff has a small error. In fact, this would not be wrong since from above we know that the error is less than 0.6%. If we set α as the weight we put on, the theoretical payoff of the strategy at maturity would be

$$CorrelSwap + \alpha \times Disp = (\rho_{real} - K) + \alpha \times \frac{1}{2K_I}\left[\left(\sum_{i=1}^{N}\omega_i\sigma_i\right)^2(\rho_{impli} - \rho_{real})\right]$$

$$= (\rho_{real} - \rho_{impli} + 0.1) + \alpha \times \frac{1}{2K_I}\left[\left(\sum_{i=1}^{N}\omega_i\sigma_i\right)^2(\rho_{impli} - \rho_{real})\right] \text{[19]}$$

$$= (\rho_{impli} - \rho_{real}) \times \left[\alpha \times \frac{1}{2K_I}\left(\sum_{i=1}^{N}\omega_i\sigma_i\right)^2 - 1\right] + 0.1$$

From this equation, only ρ_{impli} and K_I are known on the strike date and the realised volatility and realised correlation are stochastic. Thus, we want a constant payoff whatever the realised values are at maturity. I take the α that makes $\alpha \times \frac{1}{2K_I}\left(\sum_{i=1}^{N}\omega_i\sigma_i\right)^2 - 1 = 0$ thus whatever the realised correlation is the payoff would be 0.1. Since it still contains a stochastic volatility term, I assume that implied volatility is a good approximation of the future volatility. Thus, $\alpha = \frac{2K_I}{\left(\sum_{i=1}^{N}\omega_i\sigma_i\right)^2}$ where the volatility is implied and we can rewrite it as

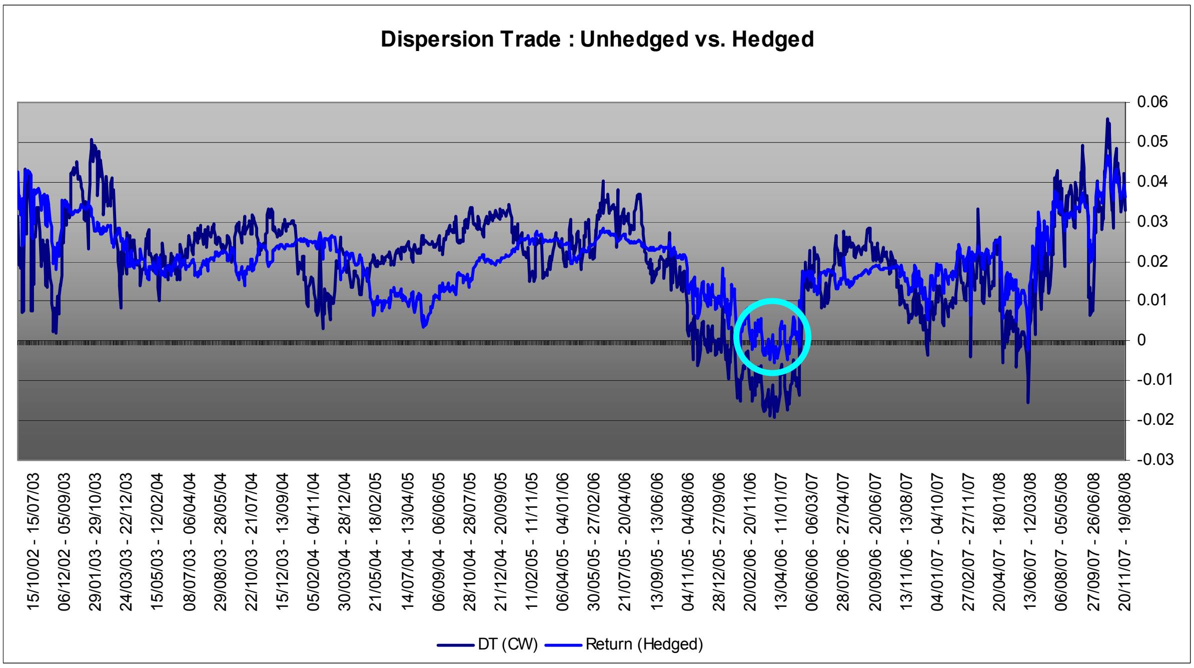
Nominal of Correlation Swap : Nominal of dispersion Trade = 1 : α

where $\alpha = \frac{2K_I}{\left(\sum_{i=1}^{N}\omega_i K_i\right)^2}$

Below is the back-test of the strategy with the new weight. We see that the average return has not been compromised; it has even increased by 0.1%. The volatility of return has decreased by 32% from the previous volatility, thus resulting in a big increase in the risk-return rate.

Dispersion Trade (Correlation Weighted)	Unhedged	Hedged
Average Return	2.6%	2.7%
Volatility of Return	1.29%	0.88%
Risk-Return Rate	2.03	3.06

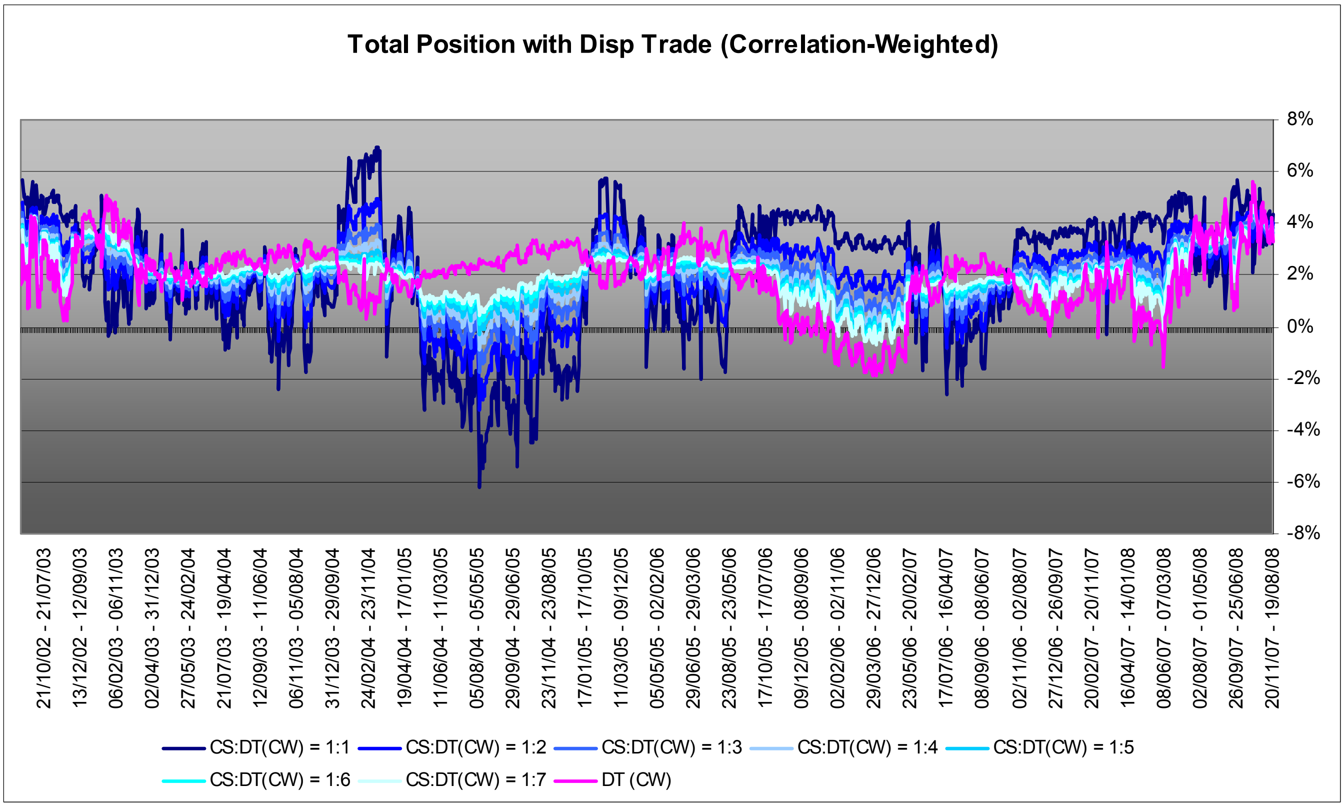
We see that in the area colored light-blue we have a negative-payoff, but it is not more than 0.5%. This is the period where the realised correlation was greater than the implied correlation so we took 'realised correlation – 0.05' as the strike. But in the hedging weight, we did not take this into account.



19 $K = \rho_{impli} - 0.1$ by hypothesis

TAILORING TO FIT THE RISK-PREFERENCE

Before, the purpose was to hedge the payoff of the dispersion trade, meaning less downside and at the same time less upside. However, there is always a trade-off between return and risk. If we want to go for more return then we have to bear more risk. In one word, it is the investor who had to choose how much risk he or she is willing to take for the return. The graph below shows the P&L of the total position with various flat weights on the nominal between the correlation swap and the dispersion trade (correlation weighted in particular).



Below is the average annualized return and volatility of the return of the strategy under the market condition of past 6 years. Also, the risk-return rate, simply the return divided by the volatility is shown below. The last two figures are to compare how effective the strategy performed taking into account the risk it was bearing. We see that as we put more weight on the dispersion trade, the average return decreases even if not by a great amount. However, the volatility of return decreases by a great amount, thus leaving us with a non-decreasing risk-return ratio. In fact, the risk-return ration increases until the weight of 1:6 and then it slightly decreases after. Thus, if we want to maximize the risk-return ratio the weight '1:6' would be optimal.

Correlation Swap : Dispersion Trade	0:1	1:1	1:2	1:3	1:4	1:5	1:6	1:7
Average (Annualized) Return	2.62%	2.76%	2.71%	2.69%	2.67%	2.67%	2.66%	2.65%
Volatility of Return	1.29%	2.40%	1.46%	1.09%	0.93%	0.88%	0.87%	0.89%
Risk-Return Ratio	2.03	1.15	1.85	2.48	2.87	3.02	3.04	2.99

APPENDIX A

Microsoft Excel - correl strat.xls

Postition	long Dispersion Trade long Corr Swap		Today	13-Aug-08
Strike Date	13-Mar-08		Correlation Strike	52.0
Maturity date	19-Dec-08		Number of Stocks	50
Date of Valorisation	31-Jul-08		Nominal	10 000 000

Press to Update

Actual Weight	Weight	Stocks and Indice	Vol Strike	Variance Cap
Crédit Agricole 0.8480 %	0.85%	ACA FP	39.29	96.47
Aegon 0.8569 %	0.86%	AGN NA	42.46	112.68
Air Liquide 0.9728 %	0.97%	AI FP	28.93	52.31
Alcatel-Lucent 0.6393 %	0.64%	ALU FP	50.29	158.06
Allianz 2.8864 %	2.89%	ALV GR	35.60	79.19
BASF 2.0405 %	2.04%	BAS GR	28.24	49.85
Bayer 1.8939 %	1.89%	BAY GR	33.55	70.35
BBVA 2.6643 %	2.66%	BBVA SM	31.41	61.67
Groupe Danone 1.2043 %	1.20%	BN FP	29.73	55.23
BNP Paribas 2.7738 %	2.77%	BNP FP	37.51	87.95
Carrefour 1.1118 %	1.11%	CA FP	29.55	54.57
AXA 2.2919 %	2.29%	CS FP	38.93	94.72
Daimler AG 3.2084 %	3.21%	DAI GR	34.51	74.44
Deutsche Börse 0.9305 %	0.93%	DB1 GR	42.22	111.41
Deutsche Bank 1.8585 %	1.86%	DBK GR	34.63	74.94
Vinci 1.0935 %	1.09%	DG FP	36.55	83.51
Deutsche Telekom 1.8084 %	1.81%	DTE GR	28.75	51.64
ENEL 1.4645 %	1.46%	ENEL IM	23.96	35.89
Eni 2.6204 %	2.62%	ENI IM	26.17	42.80
E.ON 3.7418 %	3.74%	EONGY.PK	27.42	46.99
Fortis 2.1084 %	2.11%	FORA NA	40.31	101.55
Total S.A. 5.3141 %	5.31%	FP FP	27.40	46.92
France Télécom 2.0646 %	2.06%	FTE FP	29.08	52.86
Assicurazioni Generali 1.6821 %	1.68%	G IM	25.16	39.57
Société Générale 2.2123 %	2.21%	GLE FP	42.20	111.31
Iberdrola 1.8598 %	1.86%	IBE SM	30.87	59.56
ING Group 2.8442 %	2.84%	INGA NA	39.94	99.72
Intesa Sanpaolo 2.0427 %	2.04%	ISP IM	27.62	47.67
LVMH 0.9663 %	0.97%	MC FP	32.26	65.03
ArcelorMittal 1.8906 %	1.89%	MTP FP	42.67	113.77
Munich Re 1.2433 %	1.24%	MUV2 GR	29.06	52.79
Nokia 4.6212 %	4.62%	NOK1V FH	42.35	112.12
L'Oréal 0.9810 %	0.98%	OR FP	30.64	58.69
Philips 1.3128 %	1.31%	PHIA NA	31.25	61.02
Repsol YPF 0.9028 %	0.90%	REP SM	33.04	68.23
Renault 0.9404 %	0.94%	RNO FP	42.72	114.04

contract / spot / return / average return / cov / pairwise corr / return^2 / realized var / realised vol / DT carry / payoff / corr / implied vol / implied var / vol 1M / vol 3M / vol 6M / vol 9M / parameters

APPENDIX B

```
Option Explicit


Sub GetTradeHisto_Click()

Sheet1.Calculate

If Sheet1.Cells(3, 5).Value > Sheet1.Cells(3, 10).Value Then
Sheet1.Cells(3, 5) = Sheet1.Cells(3, 10).Value
End If


Dim Answer As String


''download spot
Dim price As New SXPricesHistoric
Dim rgAdapterIn As New RangeAdapter, rgAdapterOut As New
RangeAdapter

Dim params As New SXTradeHistoricParams

With params
   .AllowExternalSource = SXTrue
   .CodeType = SXAuto
   .CompositionDateIfBasket = Date
   .PriceType = SXPriceClose
   .ProlongPercentIfBasket = 0
   .RefCurrency = SXCurr_NONE
   .UseProlongIfBasket = SXFalse

   With .HistoricParams
      .fromDate = Sheet1.Range("DATE_FROM").Value
      .toDate = Sheet1.Range("DATE_TO").Value
      .HolePolicy = SX_LeaveHoles
      .ShowHeaders = SXTrue
      .NbItemsToFill = 20
   End With
End With


Set rgAdapterIn.xlRange = Sheet1.Range(Sheet1.Range("Ric2"),
Sheet1.Range("Ric2").End(xlDown))
Set rgAdapterOut.xlRange = Sheet5.Range("G25")

price.GetTradeHistoricEx rgAdapterIn, rgAdapterOut, params

With params.HistoricParams
   .AlignedDisplayDetails = SXTrue
   .AlignedHistorics = SXTrue
  .NbItemsToFill = 20
    .HolePolicy = SX_FillHoles
End With

Set rgAdapterOut.xlRange = Sheet2.Range("A1")

price.GetTradeHistoricEx rgAdapterIn, rgAdapterOut, params

With params.HistoricParams
   .fromDate = Sheet1.Range("DATE_FROM").Value
   .toDate = Sheet1.Range("DATE_TO").Value
   .DisplayOnlyValue = SXTrue
   .NbItemsToFill = 20
End With

Set rgAdapterIn.xlRange = Sheet1.Range("UDL_NAME")
Set rgAdapterOut.xlRange = Range("L134")
price.GetTradeHistoricEx rgAdapterIn, rgAdapterOut, params

'define parameters
Dim Nbre_stocks As Integer
Dim Nbre_dates As Integer

Dim i As Integer
Dim j As Integer
Dim k As Integer

Sheet2.Activate

'detect corporate actions
For i = 1 To Nbre_stocks
   If IsEmpty(Sheet2.Cells(2, i + 1)) = True Then
      Answer = MsgBox("An error has occurred. Perhaps there has
been a corporate action. Continue?", vbYesNo)
      If Answer <> vbYes Then Exit Sub
   End If
Next i

Nbre_dates =
Application.WorksheetFunction.Count(Sheet2.Columns("A:A"))
Nbre_stocks = Sheet1.Range(Sheet1.Range("Ric"),
Sheet1.Range("Ric").End(xlDown)).Rows.Count - 2

Sheet1.Cells(3, 7) = Nbre_stocks

'fill empty cells
For i = 1 To Nbre_stocks
   If IsEmpty(Sheet2.Cells(Nbre_dates + 1, i + 1)) = True Then
      Sheet2.Cells(Nbre_dates + 1, i + 1) = Sheet2.Cells(Nbre_dates, i
+ 1)
   End If
Next i


''''calculate log return''''''''''''''''''''''''''''''''''''''''
Sheet3.Activate

For i = 0 To Nbre_stocks
   For j = 0 To Nbre_dates - 2
      If Sheet2.Cells(Nbre_dates - j, 2 + i) = Sheet2.Cells(Nbre_dates +
1 - j, 2 + i) Then
         Sheet3.Cells(Nbre_dates - j, 2 + i) = " "
      Else
         Sheet3.Cells(Nbre_dates + 1 - j, 2 + i) =
Application.WorksheetFunction.Ln(Sheet2.Cells(Nbre_dates - j, 2 +
i).Value / Sheet2.Cells(Nbre_dates + 1 - j, 2 + i).Value)
      End If
   Next j
Next i

'align
Sheet3.Activate
Sheet3.Cells.Select
Selection.Columns.AutoFit
With Selection
   .HorizontalAlignment = xlCenter
End With

''''calculate average log return''''''''''''''''''''''''''''''''''''''''
Sheet4.Activate

For i = 0 To Nbre_stocks
   For j = 0 To Nbre_dates - 2
```

```
      Sheet4.Cells(Nbre_dates + 1 - j, 2 + i) =
Application.WorksheetFunction.Average(Sheet3.Range(Sheet3.Cells(Nbre_dates + 1 - j, 2 + i), Sheet3.Cells(Nbre_dates + 1, 2 + i)))
   Next j
Next i


"""calculate return^2
Sheet10.Activate

For i = 0 To Nbre_stocks
   For j = 0 To Nbre_dates - 2
      If Sheet2.Cells(Nbre_dates - j, 2 + i) = Sheet2.Cells(Nbre_dates + 1 - j, 2 + i) Then
        Sheet10.Cells(Nbre_dates - j, 2 + i) = " "
      Else
        Sheet10.Cells(Nbre_dates + 1 - j, 2 + i) =
Sheet3.Cells(Nbre_dates + 1 - j, 2 + i) ^ 2
      End If
   Next j
Next i


""""calculate FRV^2 and realised var""""""""""""""""""""""""
'realised vol = average(rdt^2) * 252

For i = 0 To Nbre_stocks
   For j = 0 To Nbre_dates - 2
      Sheet9.Cells(Nbre_dates + 1 - j, 2 + i) =
Application.WorksheetFunction.Average(Sheet10.Range(Sheet10.Cells(Nbre_dates + 1, 2 + i), Sheet10.Cells(Nbre_dates + 1 - j, 2 + i))) * 252
   Next j
Next i



"""cov""""""""""""""""""""""""""""
' log return - average spot
Sheet6.Activate

For i = 0 To Nbre_stocks
   For j = 0 To Nbre_dates - 2
      If Sheet3.Cells(Nbre_dates + 1 - j, 2 + i) = " " Then
        Sheet6.Cells(Nbre_dates + 1 - j, 2 + i) = -
Sheet4.Cells(Nbre_dates + 1 - j, 2 + i)
      Else
      Sheet6.Cells(Nbre_dates + 1 - j, 2 + i) = Sheet3.Cells(Nbre_dates + 1 - j, 2 + i) - Sheet4.Cells(Nbre_dates + 1 - j, 2 + i)
      End If
   Next j
Next i


"'pairwise correlation""""""""""""""""""""""
Dim Nbre_comb As Integer
Nbre_comb = Nbre_stocks * (Nbre_stocks - 1) * 0.5
Dim m As Integer
Dim n As Integer
Dim Sum_cov As Double
Dim Sum_var As Double
Dim product As Double
Dim product1 As Double
Dim product2 As Double
Dim Sum_product1 As Double
Dim Sum_product2 As Double

        Sheet7.Activate

'k = 1
'For i = 1 To Nbre_stocks - 1
'
'    For m = 1 To Nbre_stocks - i
'
'       Sheet7.Cells(1 + k, 1) = Sheet6.Cells(1, 1 + i).Value & " vs. " &
Sheet6.Cells(1, 1 + i).Offset(0, m).Value
'
'       For n = 1 To Nbre_dates - 2
'
'           product1 = 0
'           product2 = 0
'           Sum_product1 = 0
'           Sum_product2 = 0
'           Sum_var = 0
'           Sum_cov = 0
'
'           For j = 0 To n
'               product = Sheet6.Cells(Nbre_dates + 1 - j, 1 + i) *
Sheet6.Cells(Nbre_dates + 1 - j, 1 + i).Offset(0, m)
'               product1 = (Sheet6.Cells(Nbre_dates + 1 - j, 1 + i)) ^ 2
'               product2 = (Sheet6.Cells(Nbre_dates + 1 - j, 1 + i).Offset(0, m)) ^ 2
'
'               Sum_cov = Sum_cov + product
'               Sum_product1 = Sum_product1 + product1
'               Sum_product2 = Sum_product2 + product2
'           Next j
'
'           Sheet7.Cells(1 + k, Nbre_dates + 2 - j) = Sum_cov /
Sqr(Sum_product1) / Sqr(Sum_product2)
'
'       Next n
'
'       k = k + 1
'    Next m
'Next i


"'page results-P&L""""""""""""""""""""""""""""""""

"'realized part""""""""""""""""""""""""""

Sheet8.Activate

'average correlation for corr swap
Sheet19.Cells(1, 2) = "Average Pairwise Corr"

For i = 1 To Nbre_dates - 2
   Sheet19.Cells(2 + i, 2) =
(Application.WorksheetFunction.Average(Sheet7.Range(Sheet7.Cells(2, 2 + i), Sheet7.Cells(Nbre_comb + 1, 2 + i)))) - Sheet1.Cells(2, 8).Value / 100 '* (Sheet8.Cells(2 + i, 1) - Sheet1.Cells(2, 5)) / (Sheet1.Cells(3, 10) - Sheet1.Cells(1, 10))
Next i

"'P&L of Corr Swap
Sheet8.Cells(1, 2) = "P&L (CS)"

For i = 1 To Nbre_dates - 2
   Sheet8.Cells(2 + i, 2) = Sheet19.Cells(2 + i, 2) - Sheet1.Cells(2, 7).Value / 100
Next i


""'Carry of Dispersion""""""""""""""""""""""""""""
'(realised var - (vol/100)^2 ) / (2 * vol strike /100) * time elapsed
Sheet16.Activate

For j = 1 To Nbre_dates - 1
   For i = 1 To Nbre_stocks
      Sheet16.Cells(2 + j, i + 1) = (Sheet9.Cells(2 + j, i + 1) -
(Sheet1.Cells(7 + i, 5) / 100) ^ 2) / (2 * Sheet1.Cells(7 + i, 5) / 100) '_
                        '* (Sheet12.Cells(2 + j, 1) - Sheet1.Cells(1, 10)) /
(Sheet1.Cells(3, 10) - Sheet1.Cells(1, 10))
   Next i
      Sheet16.Cells(2 + j, Nbre_stocks + 2) = (-Sheet9.Cells(2 + j,
Nbre_stocks + 2) + (Sheet1.Cells(7 + i, 5) / 100) ^ 2) / (2 *
Sheet1.Cells(7 + i, 5) / 100) '_
                        '* (Sheet12.Cells(2 + j, 1) - Sheet1.Cells(1, 10)) /
(Sheet1.Cells(3, 10) - Sheet1.Cells(1, 10))
Next j

"to calculate sumproduct after
Sheet1.Range(Cells(8, 3), Cells(7 + Nbre_stocks, 3)).Copy
Sheet5.Range("C3").PasteSpecial Paste:=xlPasteAll,
Operation:=xlNone, SkipBlanks:= _
```

```
False, Transpose:=True
'to calculate correlation
For i = 0 To Nbre_stocks - 1
      Sheet5.Cells(4, 3 + i) = Sheet5.Cells(3, 3 + i) ^ 2
Next i

'''carry of dispersion''''''
Sheet8.Activate

'weighted carry
Sheet8.Cells(1, 3) = "P&L (DT - basket)"
Sheet8.Cells(1, 4) = "P&L (DT - index)"

For i = 1 To Nbre_dates - 1
Sheet8.Cells(2 + i, 3) =
Application.WorksheetFunction.SumProduct(Sheet5.Range(Sheet5.Cells(3, 3), Sheet5.Cells(3, Nbre_stocks + 2)), _
                  Sheet16.Range(Sheet16.Cells(2 + i, 2), Sheet16.Cells(2 + i, Nbre_stocks + 1)))
Sheet8.Cells(2 + i, 4) = Sheet16.Cells(2 + i, Nbre_stocks + 2)
Next i

'realized part
Sheet8.Cells(1, 5) = "P&L (DT)"

For i = 1 To Nbre_dates - 1
   Sheet8.Cells(2 + i, 5) = Sheet8.Cells(2 + i, 3) + Sheet8.Cells(2 + i, 4)
Next i

'''P&L of total position
Sheet8.Cells(1, 6) = "P&L (Total)"

For i = 1 To Nbre_dates - 2
   Sheet8.Cells(2 + i, 6) = Sheet8.Cells(2 + i, 2) + Sheet8.Cells(2 + i, 5)
Next i



        ''''''''correlation''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''
'''realised vol - to calculate realised corr
Sheet17.Activate

For j = 1 To Nbre_dates - 1
   For i = 1 To Nbre_stocks + 1
      Sheet17.Cells(2 + j, i + 1) = Sqr(Sheet9.Cells(2 + j, i + 1))
   Next i
Next j


'''average realised volatility''''''''''''''''''''''
Sheet19.Activate

Sheet19.Cells(1, 9) = "Average Realised Volatility"

For i = 2 To Nbre_dates
   Sheet19.Cells(1 + i, 9) =
Application.WorksheetFunction.Average(Sheet17.Range(Sheet17.Cells(1 + i, 2), Sheet17.Cells(1 + i, 1 + Nbre_stocks)))
Next i


'''Dirty realised Correlation
Sheet19.Activate
Sheet19.Cells(1, 3) = "Dirty Realised Corr"

For i = 1 To Nbre_dates - 1
   Sheet19.Cells(2 + i, 3) = Sheet9.Cells(2 + i, Nbre_stocks + 2) /
Application.WorksheetFunction.SumProduct _
                        (Sheet5.Range(Sheet5.Cells(3, 3), Sheet5.Cells(3, Nbre_stocks + 2)), _
                        Sheet17.Range(Sheet17.Cells(2 + i, 2), Sheet17.Cells(2 + i, Nbre_stocks + 1))) ^ 2
Next i

'''clean realised correlation
Sheet19.Cells(1, 4) = "Clean Realised Corr"
For i = 1 To Nbre_dates - 1
   Sheet19.Cells(2 + i, 4) = (Sheet9.Cells(2 + i, Nbre_stocks + 2) -
Application.WorksheetFunction.SumProduct _
                        (Sheet5.Range(Sheet5.Cells(4, 3), Sheet5.Cells(4, Nbre_stocks + 2)), _
                        Sheet9.Range(Sheet9.Cells(2 + i, 2), Sheet9.Cells(2 + i, Nbre_stocks + 1)))) _
                        / (Application.WorksheetFunction.SumProduct _
                        (Sheet5.Range(Sheet5.Cells(3, 3), Sheet5.Cells(3, Nbre_stocks + 2)), _
                        Sheet17.Range(Sheet17.Cells(2 + i, 2), Sheet17.Cells(2 + i, Nbre_stocks + 1))) ^ 2 _
                        - Application.WorksheetFunction.SumProduct _
                        (Sheet5.Range(Sheet5.Cells(4, 3), Sheet5.Cells(4, Nbre_stocks + 2)), _
                         Sheet9.Range(Sheet9.Cells(2 + i, 2), Sheet9.Cells(2 + i, Nbre_stocks + 1))))
Next i


'''fill holes in implied volatility
'1M
Sheet12.Activate

For j = 1 To Nbre_stocks + 1
   For i = 1 To Nbre_dates
      If IsEmpty(Sheet12.Cells(Nbre_dates + 1 - i, 1 + j)) = True Then
        Sheet12.Cells(Nbre_dates + 1 - i, 1 + j) = Sheet12.Cells(Nbre_dates + 2 - i, 1 + j)
      End If
   Next i
Next j

'3M
Sheet13.Activate

For j = 1 To Nbre_stocks + 1
   For i = 1 To Nbre_dates
      If IsEmpty(Sheet13.Cells(Nbre_dates + 1 - i, 1 + j)) = True Then
        Sheet13.Cells(Nbre_dates + 1 - i, 1 + j) = Sheet13.Cells(Nbre_dates + 2 - i, 1 + j)
      End If
   Next i
Next j

'6M
Sheet14.Activate

For j = 1 To Nbre_stocks + 1
   For i = 1 To Nbre_dates
      If IsEmpty(Sheet14.Cells(Nbre_dates + 1 - i, 1 + j)) = True Then
        Sheet14.Cells(Nbre_dates + 1 - i, 1 + j) = Sheet14.Cells(Nbre_dates + 2 - i, 1 + j)
      End If
   Next i
Next j

'9M
Sheet15.Activate

For j = 1 To Nbre_stocks + 1
   For i = 1 To Nbre_dates
      If IsEmpty(Sheet15.Cells(Nbre_dates + 1 - i, 1 + j)) = True Then
        Sheet15.Cells(Nbre_dates + 1 - i, 1 + j) = Sheet15.Cells(Nbre_dates + 2 - i, 1 + j)
      End If
   Next i
Next j

' '''implied volatility''''''''''''''''''''''''''''''''''''''''
Sheet20.Activate

For i = 0 To Nbre_dates - 1
   For j = 1 To Nbre_stocks + 1

      If Sheet20.Cells(Nbre_dates + 1 - i, 1).Value <= Sheet1.[maturity].Value - 273 Then
```

```
        Sheet20.Cells(Nbre_dates + 1 - i, 1 + j) =
Sheet15.Cells(Nbre_dates + 1 - i, 1 + j) / 100

      ElseIf Sheet20.Cells(Nbre_dates + 1 - i, 1).Value >
Sheet1.[maturity].Value - 273 And _
        Sheet20.Cells(Nbre_dates + 1 - i, 1).Value <=
Sheet1.[maturity].Value - 182 Then
        Sheet20.Cells(Nbre_dates + 1 - i, 1 + j) =
(Sheet14.Cells(Nbre_dates + 1 - i, 1 + j) + _
          (Sheet15.Cells(Nbre_dates + 1 - i, 1 + j) -
Sheet14.Cells(Nbre_dates + 1 - i, 1 + j)) * _
          (Sheet1.[maturity] - Sheet20.Cells(Nbre_dates + 1 - i,
1).Value - 182) / 91) / 100

      ElseIf Sheet20.Cells(Nbre_dates + 1 - i, 1).Value >
Sheet1.[maturity].Value - 182 And _
        Sheet20.Cells(Nbre_dates + 1 - i, 1).Value <=
Sheet1.[maturity].Value - 91 Then
        Sheet20.Cells(Nbre_dates + 1 - i, 1 + j) =
(Sheet13.Cells(Nbre_dates + 1 - i, 1 + j) + _
          (Sheet14.Cells(Nbre_dates + 1 - i, 1 + j) -
Sheet13.Cells(Nbre_dates + 1 - i, 1 + j)) * _
          (Sheet1.[maturity] - Sheet20.Cells(Nbre_dates + 1 - i,
1).Value - 91) / 91) / 100

      ElseIf Sheet20.Cells(Nbre_dates + 1 - i, 1).Value >
Sheet1.[maturity].Value - 91 And _
        Sheet20.Cells(Nbre_dates + 1 - i, 1).Value <=
Sheet1.[maturity].Value - 30 Then
        Sheet20.Cells(Nbre_dates + 1 - i, 1 + j) =
(Sheet12.Cells(Nbre_dates + 1 - i, 1 + j) + _
          (Sheet13.Cells(Nbre_dates + 1 - i, 1 + j) -
Sheet12.Cells(Nbre_dates + 1 - i, 1 + j)) * _
          (Sheet1.[maturity] - Sheet20.Cells(Nbre_dates + 1 - i,
1).Value - 91) / 61) / 100

      ElseIf Sheet20.Cells(Nbre_dates + 1 - i, 1).Value >
Sheet1.[maturity].Value - 30 Then
        Sheet20.Cells(Nbre_dates + 1 - i, 1 + j) =
Sheet12.Cells(Nbre_dates + 1 - i, 1 + j) / 100

      End If

    Next j
Next i


''''''implied correlation''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''
'''implied var - to calculate implied corr
Sheet21.Activate

For j = 1 To Nbre_dates - 1
    For i = 1 To Nbre_stocks + 1
      Sheet21.Cells(2 + j, i + 1) = Sheet20.Cells(2 + j, i + 1) ^ 2
    Next i
Next j

'''Dirty Implied Correlation
Sheet19.Activate

Sheet19.Cells(1, 6) = "Dirty Implied Correlation"

For i = 1 To Nbre_dates - 1
    Sheet19.Cells(2 + i, 6) = Sheet21.Cells(2 + i, Nbre_stocks + 2) /
(Application.WorksheetFunction.SumProduct _
                  (Sheet5.Range(Sheet5.Cells(3, 3), Sheet5.Cells(3,
Nbre_stocks + 2)), _
                  Sheet20.Range(Sheet20.Cells(2 + i, 2),
Sheet20.Cells(2 + i, Nbre_stocks + 1)))) ^ 2
Next i

'''Clean Implied correlation
Sheet19.Cells(1, 7) = "Clean Implied Correlation"

For i = 1 To Nbre_dates - 1
    Sheet19.Cells(2 + i, 7) = (Sheet21.Cells(2 + i, Nbre_stocks + 2) -
Application.WorksheetFunction.SumProduct _
                  (Sheet5.Range(Sheet5.Cells(4, 3), Sheet5.Cells(4,
Nbre_stocks + 2)), _
                  Sheet21.Range(Sheet21.Cells(2 + i, 2),
Sheet21.Cells(2 + i, Nbre_stocks + 1)))) _
                  / (Application.WorksheetFunction.SumProduct _
                  (Sheet5.Range(Sheet5.Cells(3, 3), Sheet5.Cells(3,
Nbre_stocks + 2)), _
                  Sheet20.Range(Sheet20.Cells(2 + i, 2),
Sheet20.Cells(2 + i, Nbre_stocks + 1))) ^ 2 _
                  - Application.WorksheetFunction.SumProduct _
                  (Sheet5.Range(Sheet5.Cells(4, 3), Sheet5.Cells(4,
Nbre_stocks + 2)), _
                   Sheet21.Range(Sheet21.Cells(2 + i, 2),
Sheet21.Cells(2 + i, Nbre_stocks + 1))))
Next i


'''DT theoretical value''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''
Sheet8.Activate

Sheet8.Cells(1, 8) = "P&L (DT-Theoretical)"

For i = 2 To Nbre_dates - 1
    Sheet8.Cells(1 + i, 8) = Application.WorksheetFunction.SumProduct
_
                  (Sheet5.Range(Sheet5.Cells(3, 3), Sheet5.Cells(3,
Nbre_stocks + 2)), _
                  Sheet17.Range(Sheet17.Cells(2 + i, 2),
Sheet17.Cells(2 + i, Nbre_stocks + 1))) ^ 2 _
                  * (Sheet19.Cells(1 + i, 7) - Sheet19.Cells(1 + i, 4)) / (2
* Sheet1.[Ric].Offset(Nbre_stocks + 1, 1) / 100)
Next i



'''copy MtM from DashBoard
'Sheet8.Cells(1, 3) = "MtM"
'
'Dim chemin As String
'Dim onglet As String
'Dim Nbre_datesM As Integer
'
'onglet = Sheet1.[onglet]
'chemin = Sheet1.[chemin] & "\" & Sheet1.[fichier]
'
'Workbooks.Open Filename:=chemin, UpdateLinks:=0, ReadOnly:=1
'Sheets(onglet).Select
'
'Nbre_datesM = Sheets(onglet).Range(Sheets(onglet).Cells(9, 2),
Sheets(onglet).Cells(9, 2).End(xlDown)).Rows.Count
'
'For i = 1 To Nbre_dates - 1
'    For j = 1 To Nbre_datesM
'        If Sheet8.Cells(i + 1, 1) = Sheets(onglet).Cells(8, 2).Offset(j,
0).Value Then
'            If IsEmpty(Sheets(onglet).Cells(8, 2).Offset(j, 2)) = True Then
'                Sheet8.Cells(i + 1, 3) = " "
'            Else
'                Sheet8.Cells(i + 1, 3) = Sheets(onglet).Cells(8, 2).Offset(j,
2).Value
'            End If
'        End If
'    Next j
'Next i
'
'ActiveWorkbook.Close False
'
''fill blanks
'For i = 1 To Nbre_dates - 1
'        If Sheet8.Cells(Nbre_dates, 2).Offset(-i, 0) = 0 Then
'            Sheet8.Cells(Nbre_dates, 2).Offset(-i, 0) =
Sheet8.Cells(Nbre_dates, 2).Offset(-i + 1, 0)
'        End If
'Next i
'
'For i = 1 To Nbre_dates - 1
'        If Sheet8.Cells(Nbre_dates, 3).Offset(-i, 0) = 0 Then
```

```
'           Sheet8.Cells(Nbre_dates, 3).Offset(-i, 0) =
Sheet8.Cells(Nbre_dates, 3).Offset(-i + 1, 0)
'       End If
'Next i
'
''''calculate implied part''''''''''''''''''''''''''
'Sheet8.Cells(1, 4) = "Implied Part"
'
'For i = 1 To Nbre_dates - 1
'    Sheet8.Cells(i + 1, 4) = Sheet8.Cells(i + 1, 3) - Sheet8.Cells(i + 1, 2)
'Next i
'
'
''''realized part, MtM, implied part in Euros
'Sheet8.Cells(1, 5) = "Realized Part" & Chr(10) & "(EUR)"
'Sheet8.Cells(1, 6) = "MtM" & Chr(10) & "(EUR)"
'Sheet8.Cells(1, 7) = "Implied Part" & Chr(10) & "(EUR)"
'
'For i = 1 To Nbre_dates - 1
'    Sheet8.Cells(1 + i, 5) = Sheet8.Cells(1 + i, 2) * Sheet1.Cells(4,
8).Value
'    Sheet8.Cells(1 + i, 6) = Sheet8.Cells(1 + i, 3) * Sheet1.Cells(4,
8).Value
'    Sheet8.Cells(1 + i, 7) = Sheet8.Cells(1 + i, 4) * Sheet1.Cells(4,
8).Value
'Next i
'

End Sub
```

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