Price impact of order book events

Rama CONT†, Arseniy KUKANOV†, Sasha STOIKOV‡

†IEOR Department, Columbia University
‡School of ORIE, Cornell University

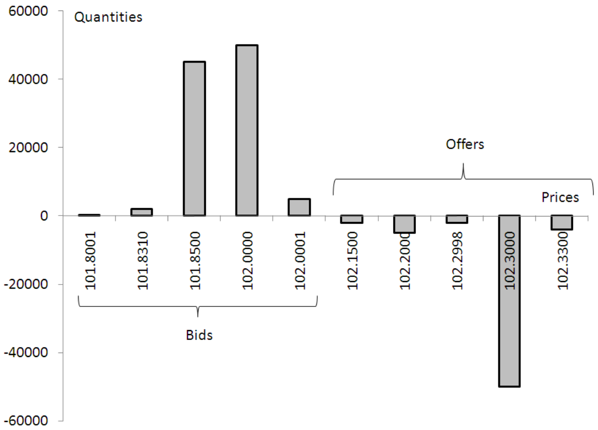
Motivation

- A primary concern in high frequency trading, trade execution and market design is *price impact*: the impact of supply/ demand on prices needs to be modeled in a realistic way.
- Order placement strategies, both in theory and in practice crucially depend on the price impact assumptions [Bertsimas, Lo '98], [Almgren, Chriss '01], [Obizhaeva, Wang '05].
- Various price impact models exist in the literature but there is little agreement on how to model it.
- Empirical studies have focused on *transaction data* and *trading volume* - [Karpoff '87], [Hasbrouck '91], [Hausman et al. '92], [Engle and Russel '98], [Lillo et al. '03], [Bouchaud et al. '08] - to name a few.
- At intraday frequencies, most (sometimes $> 95\%$) of orders are canceled before execution so the study of *order book events* provides a far more complete picture of the high frequency dynamics of supply and demand in order-driven markets.

Motivation

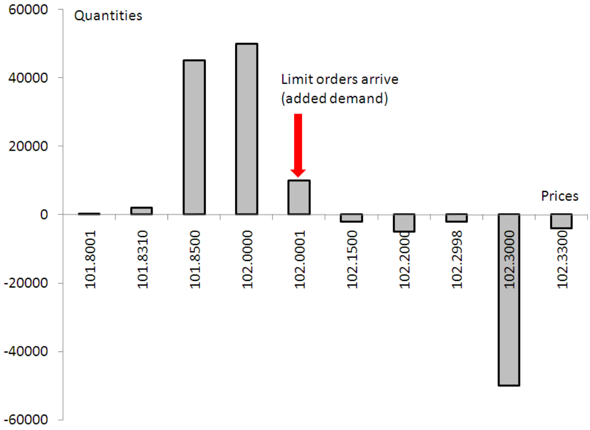
- The recent changes in structure of liquid markets, and the wide spread of algorithmic trading across the world fosters competition and innovation in order execution services.
- The new generation of execution algorithms does not rely solely on historical transactions data and increasingly uses limit orders, creating a need for more detailed models of price impact and price formation in limit order markets.
- Recent studies have showed that limit orders and cancelations, not just trades, have a tangible effect on prices - [Hautsch and Huang '09], [Bouchaud et al. '09].
- Our result: a parsimonious model relating price changes, order book events and market depth.

Limit order book



In an order-driven market, the current state of supply and demand is summarized by the **limit order book**. What can happen next?

Limit order book



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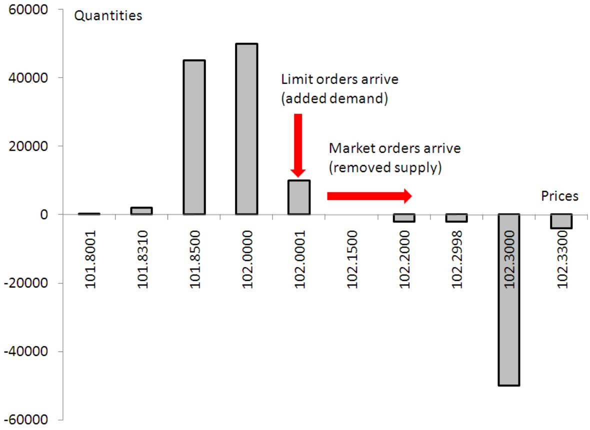
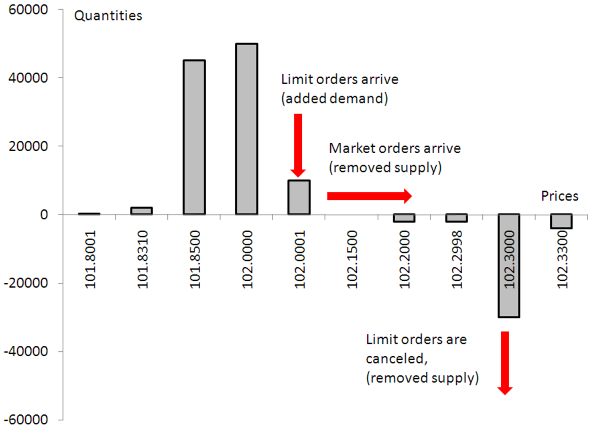


Limit order book



Price dynamics in a block order book

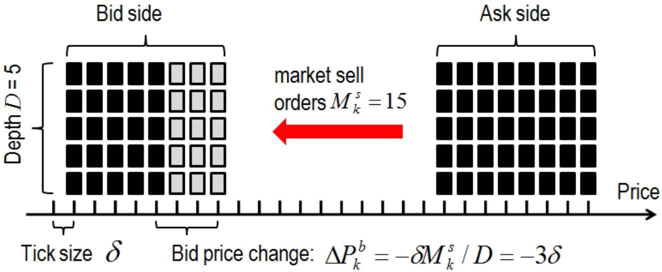
Full order book dynamics are intractable and not necessarily relevant (more on that later). Therefore, we focus on the best bid and ask quotes.

To understand the relation between order flow and price changes, consider a *discrete block order book* where:

- Limit orders arrivals and cancelations occur only at the best bid/ask.
- The number of shares at each price level beyond the bid/ask is equal to D.
- When an order queue at the bid (ask) quote grows to D, the next limit order arrives one tick above (below) that quote.

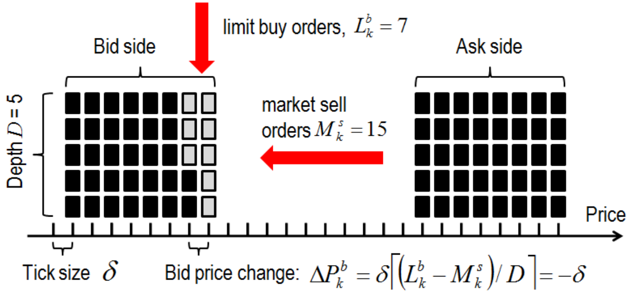
Price dynamics in a block order book

During the time $[t_{k-1}, t_k]$, market orders remove M_k^s shares from the bid.



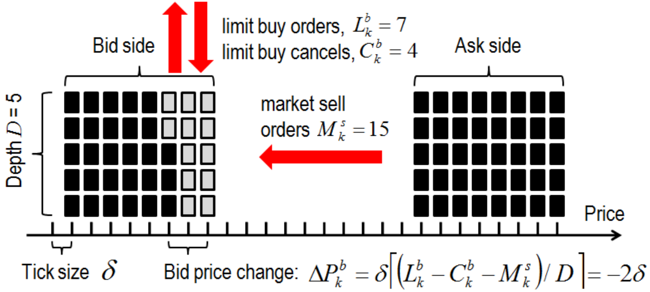
Price dynamics in a block order book

Besides that, limit orders add L_k^b shares to the bid at the same time.



Price dynamics in a block order book

Finally, cancelations remove C_k^b shares from the bid.



Price dynamics in a block order book

In this stylized order book, the impact of limit orders, market orders and cancelations on bid and ask prices is *additive* and only depends on their net imbalance:

$$\Delta P_k^b = \delta \left[\frac{L_k^b - C_k^b - M_k^s}{D} \right]$$

$$\Delta P_k^s = -\delta \left[\frac{L_k^s - C_k^s - M_k^b}{D} \right],$$

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For the mid-price changes ΔP_k we can write an even simpler relation:

$$\Delta P_k = \frac{OFI_k}{2D} + \epsilon_k$$

where $OFI_k = L_k^b - C_k^b - M_k^s - L_k^s + C_k^s + M_k^b$ and ϵ_k is the truncation error. This relations is remarkably simple - it involves no parameters and connects price dynamics, order flow and market depth.

Price impact model

- Motivated by the price impact equation in the stylized model:

$$\Delta P_k = \frac{OFI_k}{2D} + \epsilon_k, \tag{1}$$

we assume a relation between two vairables holds locally for $[t_{k-1}, t_k] \subset [T_{i-1}, T_i]$:

$$\Delta P_{k,i} = \beta_i OFI_{k,i} + \epsilon_{k,i}, \tag{2}$$

where β_i is the *price impact coefficient* and $\epsilon_{k,i}$ is a noise term.

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- The block order book model suggests that β_i is inversely proportional to the order book depth during $[T_{i-1}, T_i]$:

$$\beta_i = \frac{c}{D_i^\lambda} + \nu_i,$$

- If (1) were actually true, we would have $c = \frac{1}{2}, \ \lambda = 1$.

Data

Main dataset:

- One calendar month (April, 2010) of quotes and trades data for 50 U.S. stocks (selected at random from the S&P 500 components).
- Source: TAQ Consolidated Quotes and TAQ Consolidated Trades.
- We construct NBBO quotes from TAQ data.
- We match NBBO quotes and trades by comparing bid/ask and trade prices (so-called quote test) and we filter out large bid-ask spreads.
- Timescales $T_i - T_{i-1} = 30$ minutes, $t_k - t_{k-1} = 10$ seconds.

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Auxilary dataset:

- One month of NASDAQ ITCH messages for a representative stock from the main dataset (Schlumberger).
- Source: LOBSTER website.
- Contains all order messages and timestamps up to a millisecond.

The price impact coefficient

We estimate the following regression for each half-hour subsample (indexed by i):

$$\Delta P_{k,i} = \hat{\alpha}_i + \hat{\beta}_i OFI_{k,i} + \hat{\epsilon}_{k,i} \tag{3}$$

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Here are the average results across time and stocks:

R^2	$\hat{\alpha}_i$	$\hat{\beta}_i$	$t(\hat{\alpha}_i = 0)$	$t(\hat{\beta}_i = 0)$
65%	0.0002	0.0398	-0.02	16.27

- $\hat{\alpha}_i$ is mostly insignificant, $\hat{\beta}_i$ is significant in 98% of cases.
- The relationship is linear (higher-order terms are insignificant).
- The R^2 is surprisingly high!

Price changes and order flow imbalance

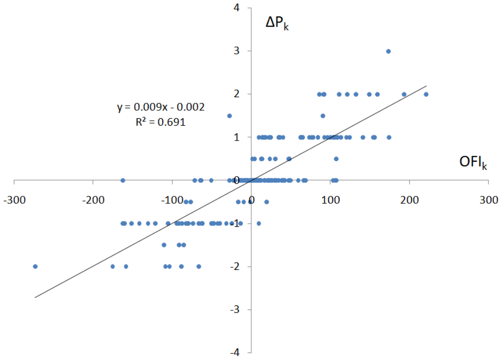

Figure: Scatter plot of $\Delta P_{k,i}$ against $OFI_{k,i}$, demonstrating a *linear* relationship between them. Stock: Schlumberger, subsample: 04/01/2010 11:30-12:00pm.

Distribution of R^2 across stocks and time

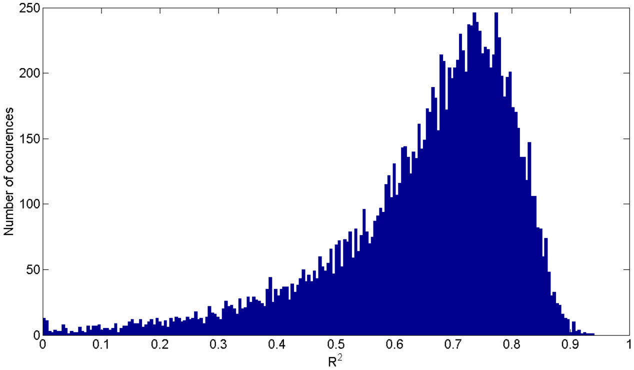

Figure: The fit is good across stocks and time

Connection with market depth

We estimate c, λ in $\beta_i = \frac{c}{D_i^\lambda} + \nu_i$ using the following regressions:

$$\log \hat{\beta}_i = \hat{\alpha}_{L,i} - \hat{\lambda} \log D_i + \hat{\epsilon}_{L,i}$$

$$\hat{\beta}_i = \hat{\alpha}_{M,i} + \frac{\hat{c}}{D_i^{\hat{\lambda}}} + \hat{\epsilon}_{M,i}$$

where $\hat{\beta}_i$ are the estimates from our previous regressions and D_i is the average of bid and ask queue sizes during the i-th interval.

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R^2	$\hat{c}$	$\hat{\lambda}$	$t(\hat{c} = 0)$	$t(\hat{\lambda} = 0)$	$t(\hat{\lambda} = 1)$
74%	0.45	0.98	20.74	29.53	-0.47

- Estimates of λ are very close to 1.
- Good fit: $R^2 \geq 50\%$ for $44/50$ stocks.
- $\lambda = 1$ is a reasonable approximation, but c varies.
- Observed depth D_i and *implied depth* $\frac{1}{2\beta_i}$ are different across stocks.

Price impact and market depth

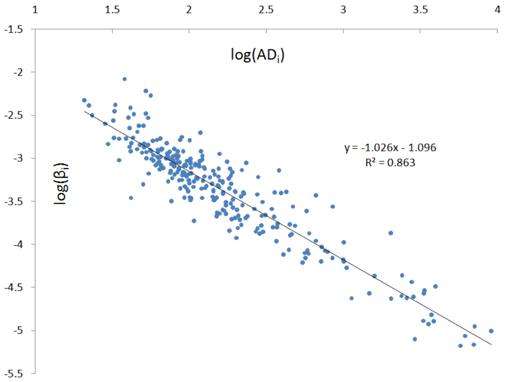

Figure: Log-log scatter plot of $\hat{\beta}_i$ against D_i, demonstrating an inverse relationship with the power -1. Stock: Schlumberger.

Distribution of $\hat{\lambda}$ across stocks

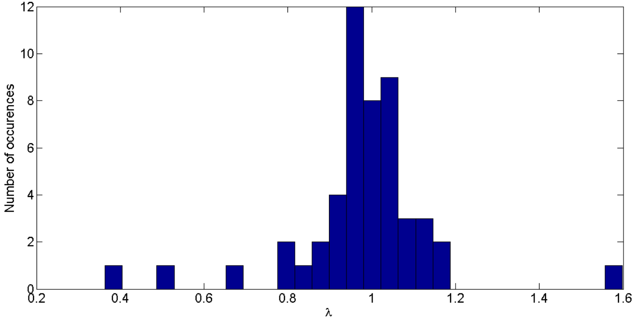

Figure: Estimates confirm the inverse relation $\beta_i = c/D_i$ implied by the block order book model

Distribution of $\hat{c}$ across stocks

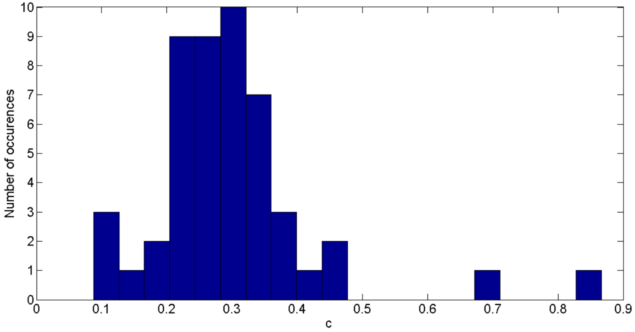

Figure: However, the constant c appears to be different from $1/2$

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- The slope β_i is exactly inversely related to market depth: $\beta_i = \frac{c}{D_i}$, but observed depth and implied depth are different.
- The linear relation between order flow imbalance and price changes can be interpreted as a simple model of instantaneous price impact that captures the effect of limit order arrivals, market orders and cancellations.

Adverse selection measurement

Price changes are relatively infrequent events on time scales that are involved in modern high-frequency trading applications. Our results show that it is possible to interpolate market dynamics between consecutive price moves with *OFI*, which tracks build-ups and depletions of order queues.

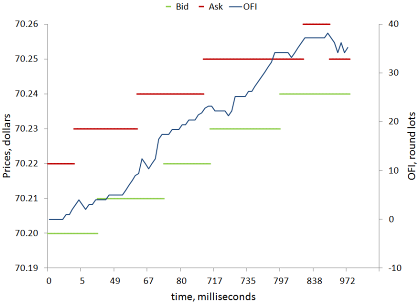

Figure: Price dynamics and cumulative *OFI* on NASDAQ for a 1-second time interval starting at 11:16:39.515 on 04/28/2010, Schlumberger stock (SLB).

Adverse selection measurement

- High positive (negative) *OFI* without a price change suggests that prices are about to go up (down). Moreover, *OFI* is positively autocorrelated over very short time intervals. This suggests using *OFI* as a measure of adverse selection in limit order executions.
- We consider every limit order execution in our auxilary dataset and estimate predictive regressions:

$$\Delta P^{post}_{k,i} = \alpha^p_i + \beta^p_i OFI^{pre}_{k,i} + \epsilon^p_{k,i},$$

where $\Delta P^{post}_{k,i}$ is the change in mid-price between t_k (k-th execution time) and $t_k + 200$ milliseconds, and $OFI^{pre}_{k,i}$ is the pre-execution order flow imbalance, computed from bid/ask queue fluctuations in $[t_k - 200, t_k - 1]$ milliseconds.

- The average R^2 of these regressions across a month is 2.93%, the average t-statistic of β^p_i is 2.68 and this coefficient is significant at a 5% level in 63% of subsamples. The average β^p_i is 0.0105.
- Stronger results are obtained with 50- and 100-millisecond windows.

Diurnal effects

- The relation $\beta_i = \frac{c}{D_i^\lambda} + \nu_i$ has an important implication: market depth has a deterministic intraday pattern, therefore price impact coefficients also vary during the day.

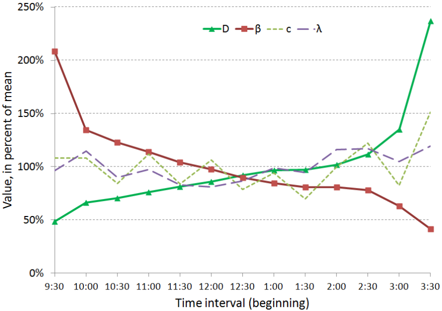

Figure: Diurnal effects in coefficients $\hat{\beta}_i, \hat{c}_i, \hat{\lambda}_i$ and depth D_i.

Diurnal effects

- Through the price impact equation, we can explain the diurnal effects in volatility:

$$\Delta P_{k,i} = \beta_i OFI_{k,i} + \epsilon_{k,i} \Rightarrow$$

$$var[\Delta P_{k,i}] = \beta_i^2 var[OFI_{k,i}] + var[\epsilon_{k,i}],$$

- As long as the variance of the residual is relatively small (i.e. R^2 is high), the volatility of prices is driven by the price impact (or depth) and the order flow variance.

Diurnal effects

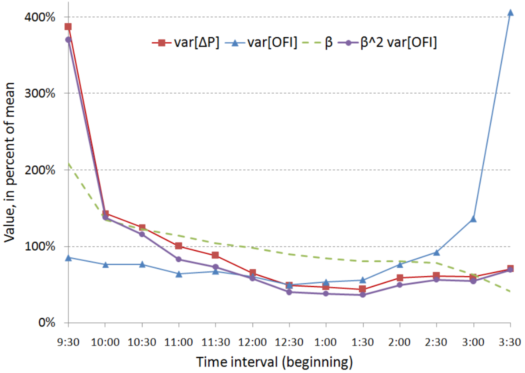

Figure: Diurnal effects in price variances $var[\Delta P_{k,i}]$ is the result of interplay between order flow variance $var[OFI_{k,i}]$ and price impact $\hat{\beta}_i$.

Volatility and volume

- "It takes volume to move the prices", but does *volume* move prices?
- The relationship between $var[\Delta P_{k,i}]$ and trading volume $VOL_{k,i}$ is widely confirmed empirically. How does this align with our model that uses $OFI_{k,i}$?
- We show that even if $\epsilon_{k,i} = 0$ (prices purely driven by *OFI* and not volume), our model *implies* a spurious relationship $var[\Delta P_{k,i}] \approx \theta_i VOL_{k,i}$, as a result of data aggregation.
- Generalization to other powers of volume is also possible.

Volatility and volume

The key is to realize that $OFI_{k,i}$ and $VOL_{k,i}$ are sums of random variables:

$$OFI_{k,i} = \sum_{n=N(t_{k-1,i})+1}^{N(t_{k,i})} e_n,$$

$$VOL_{k,i} = \sum_{n=N(t_{k-1,i})+1}^{N(t_{k,i})} w_n,$$

where $N(t_{k-1,i}) + 1$ and $N(t_{k,i})$ are the index of the first and the index of the last event in the interval $[t_{k-1}, t_k]$, e_n are (signed) order sizes and w_n are trade sizes.
Sums of random variables (under some conditions) scale according to Central Limit Theorems.

Volatility and volume

Proposition

1. *If the number of order book events grows asymptotically linearly with time* $\frac{N(\Delta t)}{\Delta t} \to \Lambda$, *as* $\Delta t \to \infty$, *where* Λ *is a constant,*
2. *Individual event contributions* e_n *are IID with finite variance* σ^2,
3. *The trade sizes* $\{w_n\}_{n=1}^{\infty}$ *are IID random variables with a finite mean* $E[w_n] = E[w_n | w_n > 0] P(\{w_n > 0\}) = \mu\pi$

Then, the following scaling relationship holds:

$$\frac{\sqrt{\mu\pi}}{\sigma} \frac{OFI(\Delta t)}{\sqrt{VOL(\Delta t)}} \Rightarrow \xi, \text{ as } \Delta t \to \infty \tag{4}$$

where ξ *is a standard normal random variable.*

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Using limit (4) as an equality, we have $var[OFI_{k,i}] \approx \frac{\sigma^2}{\mu\pi} VOL_{k,i}$, but we also have $var[\Delta P_{k,i}] = \beta_i^2 var[OFI_{k,i}] + var[\epsilon_{k,i}]$ in our model. So, even if $var[\epsilon_{k,i}]$ does not depend on volume, we get a spurious relation between price volatility and volume: $var[\Delta P_{k,i}] \approx \frac{\beta_i^2 \sigma^2}{\mu\pi} VOL_{k,i}$

Summary

- There is a linear relationship between price changes and the order flow imbalance. It performs well across stocks and timescales and involves a single parameter.
- This model helps to understand intraday dynamics of volatility and provides a relatively simple picture of price formation in modern limit order markets.
- Monitoring order flow imbalance can help to avoid adverse selection in limit order executions.

Effect of order flows deeper in the book

We compute variables $OFI^m, m = 2, \ldots, 5$ from m-th level queue fluctuations and fit regressions: $\Delta P_{k,i} = \hat{\alpha}_i^M + \sum_{m=1}^{M} \hat{\beta}_i^{m,M} OFI_{k,i}^m + \hat{\epsilon}_{k,i}^M, \ M = 1, \ldots, 5$

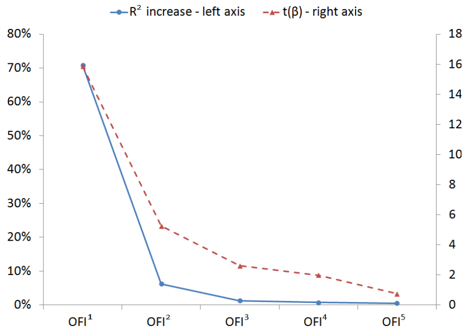

Figure: Cross-time average increase in R^2 from inclusion of $OFI^2 - OFI^5$, and their average t-statistics in the joint regression ($M = 5$).

Robustness check: timescale choice

We estimate the regression $\Delta P_{k,i} = \hat{\alpha}_i + \hat{\beta}_i OFI_{k,i} + \hat{\epsilon}_{k,i}$ for a variety of timescales $\Delta t = t_k - t_{k-1}$, ranging from 50 milliseconds to 5 minutes using separate intraday subsamples as before.

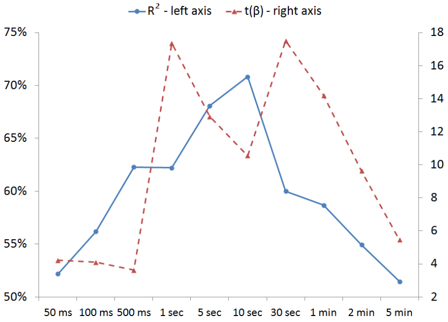

Figure: Cross-time average R^2 and t-statistics for *OFI* coefficient are high regardless of Δt.