The Alpha Engine: Designing an Automated Trading Algorithm

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Abstract

We introduce a new approach to algorithmic investment management that yields profitable automated trading strategies. This trading model design is the result of a path of investigation that was chosen nearly three decades ago. Back then, a paradigm change was proposed for the way time is defined in financial markets, based on intrinsic events. This definition lead to the uncovering of a large set of scaling laws. An additional guiding principle was found by embedding the trading model construction in an agent-base framework, inspired by the study of complex systems. This new approach to designing automated trading algorithms is a parsimonious method for building a new type of investment strategy that not only generates profits, but also provides liquidity to financial markets and does not have a priori restrictions on the amount of assets that are managed.

1 Introduction

The asset management industry is one of the largest industries in modern society. Its relevance is documented by the astonishing amount of assets that are managed. It is estimated that globally there are 64 trillion USD under management [6]. This is nearly as big as the world product of 77 trillion USD [39].

1.1 Asset Management

Asset managers use a mix of analytic methods to manage their funds. They combine different approaches from fundamental to technical analysis. The time frames range from intraday, to days and weeks, and even months. Technical analysis, a phenomenological approach, is utilized widely as a toolkit to build trading strategies.

A drawback of all such methodologies is, however, the absence of a consistent and overarching framework. What appears as a systematic approach to asset management often boils down to gut feeling, as the manager chooses from a broad blend of theories with different interpretations. For instance, the choice and configuration of indicators is subject to the specific preference of the analyst or trader. In effect, practitioners mostly apply ad hoc rules which are not embedded in a broader context. Complex phenomena such as changing liquidity levels as a function of time go unattended.

This lack of consensus, or intellectual coherence, in such a dominant and relevant industry underpinning our whole society is striking. Especially in a day and age where computational power and digital storage capacities are growing exponentially, at shrinking costs, and where there exists an abundance of machine learning algorithms and big data techniques. To illustrate, consider the recent unexpected success of Google's AlphaGo algorithm beating the best human players [11]. This is a remarkable feat for a computer, as the game of Go is notoriously complex and players often report that they select moves based solely on intuition.

There is, however, one exception in the asset management and trading industry that relies fully on algorithmic trade generation and automated execution. Referred to under the umbrella of term "high-frequency trading", this approach has witnessed substantial growth. These strategies take advantage of short-term arbitrage opportunities and typically analyse the limit order books to jump the queue, whenever there are large orders pending [10]. While high-frequency trading results in high trade volumes the assets managed with these type of strategies are around 140 billion [34]. This is microscopic compared to the size of the global assets under management.

1.2 The Foreign Exchange Market

For the development of our trading model algorithm, and the evaluation of the statistical price properties, we focus on the foreign exchange market. This market can be characterized as a complex network consisting of interacting agents: corporations, institutional and retail traders, and brokers trading through market makers, who themselves form an intricate web of interdependence. With an average daily turnover of approximately five trillion USD [8] and with price changes nearly every second, the foreign exchange market offers a unique opportunity to analyze the functioning of a highly liquid, over-the-counter market that is not constrained by specific exchange-based rules. These markets are an order of magnitude bigger than futures or equity markets [24].

In contrast to other financial markets, where asset prices are quoted in reference to specific currencies, exchange rates are symmetric: quotes are currencies in reference to other currencies. The symmetry of one currency against another neutralizes effects of trend, which are a significant drivers in other markets, such as stock markets. This property of symmetry makes currency markets notoriously hard to trade profitably.

We focus on the foreign exchange market for the development of our trading model algorithm. Its high liquidity and long/short symmetry make it an ideal environment for the research and development of fully automated and algorithmic trading strategies. Indeed, any profitable trading algorithm for this market should, in theory, also be applicable to other markets.

1.3 The Rewards and Challenges of Automated Trading

During the crisis of 2007 and 2008, the world witnessed how the financial system destabilized the real economy and destroy vast amounts of wealth. At other times, when there are favourable economic conditions, financial markets contribute to wealth accumulation. The financial system is an integral part of the real economy with a strong cross dependency. Markets are not a closed system, where the sum of all profits and losses net out. If investment strategies contribute to market liquidity, they can help stabilize prices and reduce the uncertainty in financial markets and the economy at large. For such strategies the investment returns can be viewed as a payoff for the value-added provided to the economy.

Liquid financial markets offer a large profit potential. The length of a foreign exchange price curve, as measured by the sum of up and down price movements of increments of 0.05%, during the course of a year, is, on average, approximately 1'600%, after deducting transaction costs [16]. An investor can, in theory, earn 1'600% percent unleveraged per year, assuming perfect foresight in exploiting this coastline length. With leverage, the profit potential is even greater. Obviously, as no investor has perfect foresight, capturing 1'600% is not feasible.

However, why do most investment managers have such difficulty in earning even small returns on a systematic basis, if the profit potential is so big? Especially as traders can manage their risk with sophisticated money management rules which have the potential to turn losses into profits. Again, the question arises as to why even hedge funds, who can hire the best talent in the world, find it so hard to earn consistent annual returns. For instance, the Barclay Hedge Fund Index[1], measuring the average returns of all hedge funds (except funds of funds) in their database, reports an average yearly return of 5.035% (± 4.752%) for the past four years. How can we develop liquidity-providing investment algorithms that consistently generate positive and sizable returns? What is missing in the current paradigm?

Another key criterion of the quality of an investment strategy is the size of assets that can be deployed without a deterioration of performance. Closely related to this issue is the requirement that the strategy does not distort the market dynamics. This is for example the case with the trend following strategies that are often deployed in automated trading. Such strategies have the disadvantage that the investor does not know for sure how his action of following the trend amplifies the trend. In effect, the trend follower can get locked into a position that he cannot closeout without triggering a price dislocation.

Finally, any flavour of automated trading is constrained by the current computational capacities available to researchers. Although this constraint is loosening day by day, due to the prowess of high performance computing in finance,

[1] `www.barclayhedge.com/research/indices/ghs/Hedge_Fund_Index.html`.

some approaches rely more on number crunching than others. Ideally, any trading model algorithm should be implementable with reasonable resources to make it useful and applicable in the real world.

1.4 The Hallmarks of Profitable Trading

Investment strategies need to be fully automated. For one, the number of traded instruments should not be constrained by human limitations. Then, the trading horizons should also include intraday activity, as a condition sine qua non. Complete automation has its own challenges, because computer code can go awry and cause huge damage, as witnessed by Knight Capital, which lost 500 million USD in a matter of 30 seconds due to an operational error[2].

Many modelling attempts fail, because developers succumb to curve fitting. They start with a specific data sample and tweak their model until it makes money in simulation runs. Such trading models can disappoint from the start when going live or boast good performance for some period of time until a regime shift occurs and the profitable conditions the model was optimized for disappear.

Trading models need to be parsimonious and have a limited set of variables. If the models have too many variables, the parameter space becomes vast and hard to navigate. Parsimonious models are powerful, because they are easier to calibrate, assess, and understand why they perform. Moreover, investment models need to be robust to market changes. For instance, the models can be adaptive and have their behavior depend on the current market regime. Therefore, algorithmic investment strategies have to be developed on the basis of robust and consistent approaches and methods that provide a solid framework of analysis.

Financial markets are comprised of a large number of traders that take positions on different time horizons. Agent-based models can mimic the actual traders and are therefore well suited to research market behavior [14]. If agent-based models are fractal, i.e., behave in a self-similar manner across time horizons and only differ with respect to the scaling of their parameters, the short-term models are a filter for the validity of the long-term models. In practice, this allows for the short-term agent-based models to be tested and validated over a huge data sample with a multitude of events. As a result, the scarcity of data available for the long-term model is not a hindrance of acceptance if it is self-similar with respect to the short-term models. In effect, the validation of the model structure for short-term models implies also a validation for the long-term models, by virtue of the scaling effects. In contrast, most standard modelling approaches are typically devised for one time horizon only and hence there are no self-similar models that complement each other.

Moreover, the modeling approach should be modular and enable developers to combine smaller blocks to build bigger components. In other words, models are built in a bottom up spirit, where simple building blocks are assembled into more complex units. This also implies an information flow between building blocks.

[2] `www.sec.gov/news/press-release/2013-222`.

To summarize, our aim is to develop trading models based on parsimonious, self-similar, modular, and agent-based behavior, designed for multiple time horizons and not purely driven by trend following action. The intellectual framework unifying these angles of attack is outlined in Section 3. The result of this endeavor are interacting systems that are highly dynamic, robust, and adaptive. In other words, a type of trading model that mirrors the dynamic and complex nature of financial markets. The performance of this automated trading algorithm is outlined in the next section.

In closing, it should be mentioned that transaction costs can represent real-world stumbling blocks for trading models. Investment strategies that take advantage of short-term price movements in order to achieve good performance have higher transaction volumes than longer-term strategies. This obviously increases the impact of transaction costs on the profitability. As far as possible, it is advisable to use limit orders to initiate trades. They have the advantage that the trader does not have to cross the spread to get his order executed, thus reducing or eliminating transaction costs. The disadvantage of limit orders is, however, that execution is uncertain and depends on buy and sell interest.

2 In a Nutshell: Trading Model Anatomy and Performance

In this section we provide an overview of the trading model algorithm and its performance. For all the details on the model, see Section 4 and the code that can be download from GitHub [35].

The Alpha Engine is a counter-trending trading model algorithm that provides liquidity by opening a position when markets overshoot, and manages positions by cascading and de-cascading during the evolution of the long coastline of prices, until it closes in a profit. The building blocks of the trading model are:

- an endogenous time scale called intrinsic time that dissects the price curve into directional changes and overshoots;
- patterns, called scaling laws that hold over several orders of magnitude, providing an analytical relationship between price overshoots and directional change reversals;
- coastline trading agents operating at intrinsic events, defined by the event based language;
- a probability indicator that determines the sizing of positions, by identifying periods of market activity that deviate from normal behavior;
- skewing of cascading and de-cascading designed to mitigate the accumulation of large inventory sizes during trending markets;
- the splitting of directional change and, consequently, overshoot thresholds into upwards and downwards components, i.e., the introduction of asymmetric thresholds.

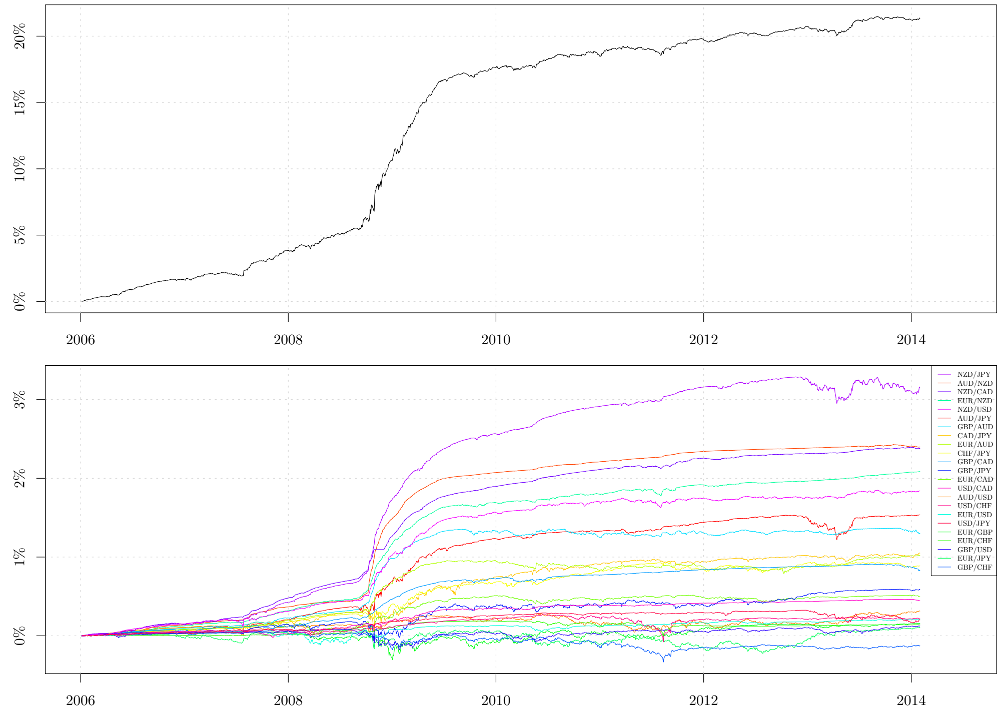

Figure 1: Daily Profit & Loss of the Alpha Engine, across 23 currency pairs, for eight years. See details in the main text of this section and Section 4.

The trading model is back-tested on historical data comprised of 23 exchange rates:

> AUD/JPY, AUD/NZD, AUD/USD, CAD/JPY, CHF/JPY, EUR/AUD, EUR/CAD, EUR/CHF, EUR/GBP, EUR/JPY, EUR/NZD, EUR/USD, GBP/AUD, GBP/CAD, GBP/CHF, GBP/JPY, GBP/USD, NZD/CAD, NZD/JPY, NZD/USD, USD/CAD, USD/CHF, USD/JPY.

The chosen time period is from the beginning of 2006 until the beginning of 2014, i.e., eight years. The trading model yields an un-levered return of 21.3401%, with an annual Sharp ratio of 3.06, and a maximum drawdown (computed on daily basis) of 0.7079%. This event occurs at the beginning of 2013 and lasts approximately 4 months, as the JYP weakens significantly following the Quantitative Easing programme ("three arrows" of fiscal stimulus) launched by the Bank of Japan.

Figure 1 shows the performance of the trading model across all exchange rates. Table B, in Appendix B, reports the monthly and yearly returns. The difference in returns among the various exchange rates is explained by volatility: the trading model reacts only to occurrences of intrinsic time events, which are functionally dependent on volatility. Exchange rates with higher volatility will have a greater number of intrinsic events and hence more opportunities for the model to extract profits from the market. This behavior can be witnessed during the financial crisis, where its deleterious effects are somewhat counterbalanced by an overall increase in profitable trading behavior of the model, fueled by the increase in volatility.

The variability in performance of the individual currency pairs can be addressed by calibrating the "aggressiveness" of the model with respect to the volatility of the exchange rate. In other words, the model trades more frequently when the volatility is low, and vice versa. For the sake of simplicity, and to avoid potential over-fitting, we have excluded these adjustments to the model. In addition, we also refrained from implementing cross-correlation measures. By assessing the behavior of the model for one currency pair, information can be gained that could be utilized as an indicator which affects the model's behaviour for other exchange rates. Finally, we have also not implemented any risk management tools.

In essence, what we present here is a proof of concept. We refrained from tweaking the model to yield better performance, in order to clearly establish and outline the model's building blocks and fundamental behavior. We strongly believe there is great potential for obvious and straightforward improvements, which would give rise to far better models. Nevertheless, the bare-bones model we present here already has the capability of being implemented as a robust and profitable trading model that can be run in real-time. With a leverage factor of 10, the model experiences a drawdown of 7.08% while yielding an average yearly profit of 10.05% for the last four years. This is still far from realizing the coastline's potential, but, in our opinion, a crucial first step in the right direction.

Finally, we conclude this section by noting that, despite conventional wisdom, it is in fact possible to "beat" a random walk. The Alpha Engine produces profitable results even on time series generated by a random walk, as seen in Figure 9 in Appendix B. This unexpected feature results from the fact that the model is dissecting Brownian motion into intrinsic time events. Now these directional changes and overshoots yield a novel context, where a cascading event is more likely to be followed by a de-cascading event than another cascading one. In detail, the probability of reaching the profitable de-cascading event after a cascade is $1 - e^{-1} \approx 0.63$, while the the probability for an additional cascade is about 0.37. In effect, the procedure of translating a tick-by-tick time series into intrinsic time events skews the odds in one's favour—for empirical as well as synthetic time series. For details see [19].

In the next section, we will embark on the journey that would ultimately result in the trading model described above. For a prehistory of events, see Appendix A.

3 Guided by an Event-Based Framework

The trading model algorithm outlined in the last section is the result of a long journey that began in the early 1980s. Starting with a new conceptual framework of time, this voyage set out to chart new terrain. The whole history of this endeavor is described in Appendix A. In the following, the key elements of this new paradigm are highlighted.

3.1 The First Step: Intrinsic Time

We all experience time as a fundamental and unshakable part of reality. In stark contrast, the philosophy of time and the notion of time in fundamental physics challenges our mundane perception of it. In an operational definition, time is simply what instruments measure and register. In this vein, we understand the passage of time in financial time series as a set of events, i.e., system interactions.

In this novel time ontology, time ceases to exist between events. In contrast to the continuity of physical time, now only interactions, or events, let a system's clock tick. Hence this new methodology is called intrinsic time [28]. This event-based approach opens the door to a modelling framework that yields self-referential behavior which does not rely on static building blocks and has a dynamic frame of reference.

Implicit in this definition is the threshold for the measurement of events. At different resolutions the same price series reveals different characteristics. In essence, intrinsic time increases the signal to noise ration in a time series by filtering out the irrelevant information between events. This dissection of price curves into events is an operator, mapping a time series $x(t)$ into a discrete set of events $\Omega[x(t), \delta]$, given the directional change threshold δ.

We focus on two types of events that represent ticks of intrinsic time:

1. a directional change δ [21, 16, 3, 5, 7];
2. an overshoot ω [16, 3, 7].

With these events, every price curve can be dissected into components that represent a change in the price trend (directional change) and a trend component (overshoot). For a directional change to be detected, first an initial direction mode needs to be chosen. As an example, in an up mode an increasing price move will result in the extremal price being updated and continuously increased. If the price goes down, the difference between the extremal price and the current price is evaluated. If this distance (in percent) exceeds the predefined directional change threshold, a directional change is registered. Now the mode is switched to down and the algorithm continues correspondingly. If now the price continues to move in the same direction as the directional change, for the size of the threshold, an overshoot event is registered. As long as a trend persists, overshoot events will be registered. See the left-hand panel in Figure 2 for an illustration. Note that two intrinsic time series will synchronize after one directional change, regardless of the chosen starting direction.

As a result, a price curve is now comprised of segments, made up of a directional change event δ and one or more overshoots of size ω. This event-based time series is called the coastline, defined for a specific directional change threshold. By measuring the various coastlines for an array of thresholds, multiple levels of event activity can be considered. See the right-hand panel in Figure 2 and Figure 3. This transformed time series is now the raw material for further

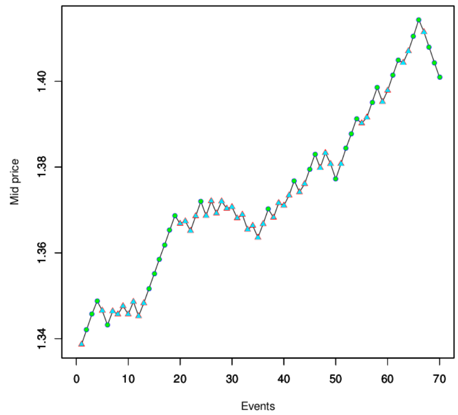

Figure 2: (*Left*) directional-change and overshoot events. (*Right*) a coastline representation of the EUR_USD price curve (2008-12-14 22:10:56 to 2008-12-16 21:58:20) defined by a directional-change threshold $\delta = 0.25\%$. The blue triangles represent directional-change and the green bullets overshoot events.

investigations [16]. In particular, this price curve will be used as input for the trading model, as described in Section 3.4. With the publication [15], the first decade came to a close.

3.2 The Emergence of Scaling Laws

A validation for the introduction of intrinsic time is that this event-based framework uncovers statistical properties otherwise not detectable in the price curves, for instance, scaling laws. Scaling-law relations characterize an immense number of natural processes, prominently in the form of

1. scaling-law distributions;
2. scale-free networks;
3. cumulative relations of stochastic processes.

Scaling-law relations display scale invariance because scaling the function's argument x preserves the shape of the function $f(x)$ [27]. Measurements of scaling-law processes yield values distributed across an enormous dynamic range, and for any section analysed, the proportion of small to large events stays constant.

Scaling-law distributions have been observed in an extraordinary wide range of natural phenomena: from physics, biology, earth and planetary sciences, economics and finance, computer science and demography to the social sciences [32, 40, 37, 31]. Although scaling-law distributions imply that small occurrences are extremely common, whereas large instances are rare, these large events

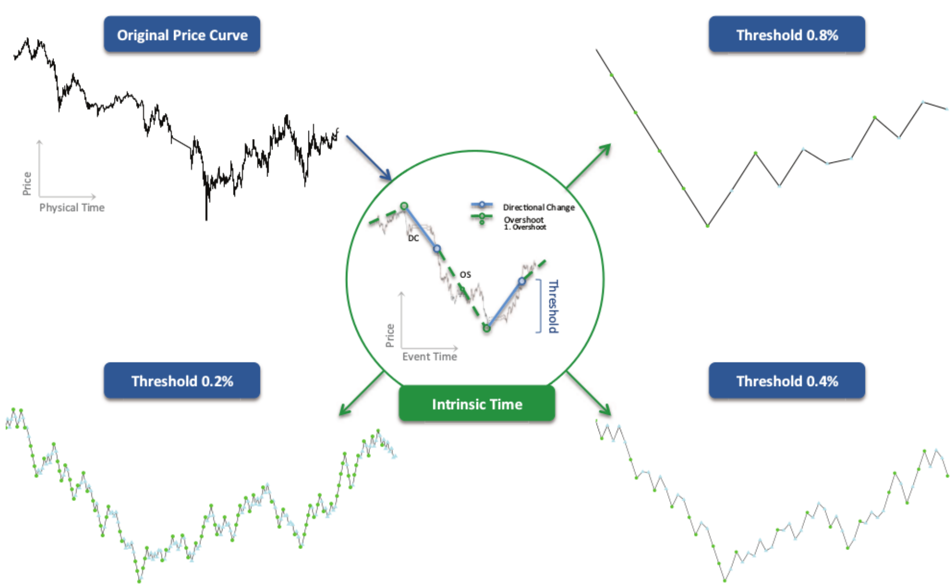

Figure 3: Coastline representation of a price curve for various directional-change thresholds δ.

occur nevertheless much more frequently compared to a normal probability distribution. Hence scaling-law distributions are said to have "fat tails".

The discovery of scale-free networks [9, 1], where the degree distributions of nodes follow a scaling-law distribution, was a seminal finding advancing the study of complex networks [30]. Scale-free networks are characterized by high robustness against random failure of nodes, but susceptible to coordinated attacks on the hubs.

Scaling-law relations also appear in collections of random variables. Prominent empirical examples are financial time-series, where one finds scaling laws governing the relationship between various observed quantities [29, 21, 15]. The introduction of the event-based framework lead to the discovery of a series of new scaling relations in the cumulative relations of properties in foreign exchange time-series [16]. In detail, of the 18 novel scaling-law relations (of which 12 are independent), 11 relate to directional changes and overshoots.

One notable observation was that, on average, a directional change δ is followed by an overshoot ω of the same magnitude

$$\langle \omega \rangle \approx \delta. \tag{1}$$

This justifies the procedure of dissecting the price curve into directional-change and overshoot segments of the same size, as seen in Figures 2 and 3. In other words, the notion of the coastline is statistically validated.

Scaling laws are a hallmark of complexity and complex systems. They can be viewed as a universal "law of nature" underlying complex behavior in all its domains.

3.3 Trading Models and Complexity

A complex system is understood as being comprised of many interacting or interconnected parts. A characteristic feature of such systems is that the whole often exhibits properties not obvious from the properties of the individual parts. This is called emergence. In other words, a key issue is how the macro behavior emerges from the interactions of the system's elements at the micro level. Moreover, complex systems also exhibit a high level of resilience, adaptability, and self-organization. The domains complex systems originate from are mostly socio-economical, biological or physio-chemical.

Complex systems are usually very reluctant to be cast into closed-form analytical expressions. This means that it is generally hard to derive mathematical quantities describing the properties and dynamics of the system under study. Nonetheless, there has been a long history of attempting to understand finance from an analytical point of view [36, 23].

In contrast, we let our trading model development be guided by the insights gained by studying complex systems [17]. The single most important feature is surprisingly subtle:

> Macroscopic complexity is the result of simple rules of interaction at the micro level.

In other words, what looks like complex behavior from a distance turns out to be the result of simple rules at closer inspection. The profundity of this observation should not be underestimated, as echoed in the words of Stephen Wolfram, when he was first struck by this realization [38, p. 9]:

> *Indeed, even some of the very simplest programs that I looked at had behavior that was as complex as anything I had ever seen. It took me more than a decade to come to terms with this result, and to realize just how fundamental and far-reaching its consequences are.*

By focusing on local rules of interactions in complex systems, the system can be naturally reduced to a set of agents and a set of functions describing the interactions between the agents. As a result, networks are the ideal formal representation of the system. Now the nodes represent the agents and the links describe their relationship or interaction. In effect, the structure of the network, i.e., its topology, determines the function of the network.

Indeed, this perspective also highlights the paradigm shift away from mathematical models towards algorithmic models, where computations and simulation are performed by computers. In other words, the analytical description of complex systems is abandoned in favor of algorithms describing the interaction of the agents. This approach has given rise to the prominent field of agent-based modeling [22, 26, 4]. The validation of agent-based models is given by their capability to replicate patterns and behavior seen in real-world complex systems by virtue of agents interacting according to simple rules.

Financial markets can be viewed as the epitome of a human-generated complex system, where the trading choices of individuals, aggregated in a market, gives rise to a stochastic and highly dynamic price evolution. In this vein, a long or short position in the market can be understood as an agent. In detail, a position p_i is comprised of the set $\{\bar{x}_i, \pm g_i\}$, where $\bar{x}_i$ is the current mid (or entry price) and $\pm g_i$ represents the position size and direction.

3.4 Coastline Trading

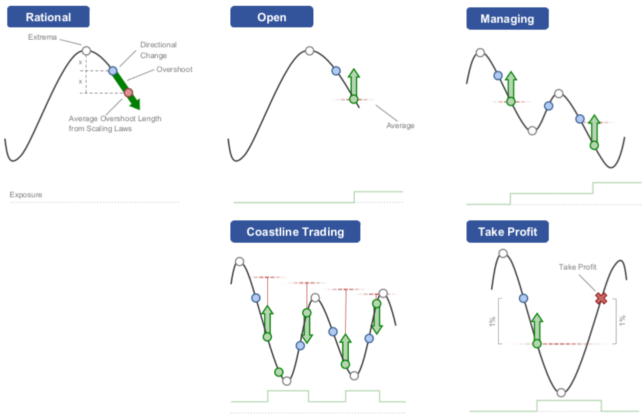

Figure 4: Simple rules: the elements of coastline trading. Cascading and de-cascading trades increase or decrease existing positions, respectively.

In a next step, we combined the event-based price curve with simple rules of interactions. This means that the agents interact with the coastline according to a set of trading rules, yielding coastline traders [18, 2, 13]. In a nutshell, the initialization of new positions and the management of existing positions in the market are clocked according to the occurrence of directional change or overshoot events. The essential elements of coastline trading are cascading and de-cascading trades. For the former, an existing position is increased by some increment in a loss, bringing the average closer to the current price. For a de-cascading event, an existing position is decreased, realizing a profit. It is important to note, that because positions sizes are only ever increased by the same fixed increments, coastline trading does not represent a Martingale strategy. In Figures 4 and 5 examples of such trading rules are shown.

With these developments, the second decade drew to a close. Led by the introduction of event-based time, uncovering scaling-law relations, the novel framework could be embedded in the larger paradigm related to the study of complex systems. The resulting trading models were, by construction, automated, agent-based, contrarian, parsimonious, adaptive, self-similar, and modular. However, there was one crucial ingredient missing, to render the models robust and hence profitable in the long-term. And so the journey continued.

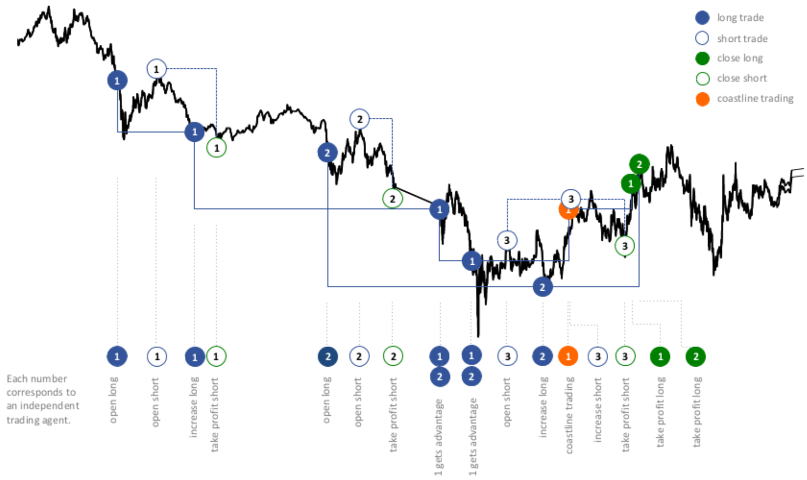

Figure 5: Real-world example of coastline trading.

3.5 Novel Insights from Information Theory

In a normal market regime, where no strong trend can be discerned, the coastline traders generate consistent returns. By construction, this trading model algorithm is attuned to directional changes and overshoots. As a result, so long as markets move in gentle fluctuations, this strategy performs. In contrast, during times of strong market trends, the agents tend to build up large positions which they cannot unload. Consequentially, each agent's inventory increases in size. As this usually happens over multiple threshold sizes, the overall resulting model behavior derails.

This challenge, related to trends, led to the incorporation of novel elements into the trading model design. A new feature, motivated by information theory was added. Specifically, a probability indicator was constructed. Equipped with this new tool, the challenges presented by market trends could now be tackled. In effect, the likeliness of a current price evolution with respect to a Brownian motion can be assessed in a quantitative manner.

In the following, we will introduce the probability indicator $\mathcal{L}$. This is an information theoretic value that measures the unlikeliness of the occurrence of price trajectories. As always, what is actually analyzed, is the price evolution which is mapped onto the discretized price curve, which results from the event-based language in combination with the overshoot scaling law. Point-wise entropy, or surprise, is defined as the entropy for a certain realization of a random variable. Following [12], we understand the surprise of the event-based price curve being related to the transitioning probability from the current state s_i to the next intrinsic event s_j, i.e., $\mathbb{P}(s_i \to s_j)$. In detail, given a directional change threshold δ, the set of possible events is given by directional changes or overshoots. In other words, a state at "time" i is given by $s_i \in \mathcal{S} = \{\delta, \omega\}$. Given $\mathcal{S}$, we now can understand all possible transitions as happening in the

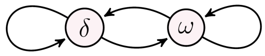

Figure 6: The transition network of states in the event-based representation of the price trajectories. Directional changes δ and overshoots ω are the building blocks of the discretized price curve, defining intrinsic time.

stylized network of states seen in Figure 6. The evolution of intrinsic time can progress from a directional change, to another directional change or an overshoot. Which, in turn, can transition to another overshoot event ω or a back to a directional change δ.

We define the surprise of the transitions from state s_i to state s_j as

$$\gamma_{ij} = -\log\mathbb{P}(s_i \to s_j), \tag{2}$$

which, as mentioned, is the point-wise entropy that is large when the probability of transitioning from state s_i to state s_j is small and vice versa. Consequently, the surprise of a price trajectory within a time interval $[0, T]$, that has experienced K transitions, is

$$\gamma_K^{[0,T]} = \sum_{k=1}^{K} -\log\mathbb{P}(s_{i_k} \to s_{i_{k+1}}). \tag{3}$$

This is now a measure of the unlikeliness of price trajectories. It is a path dependent measurement: two price trajectories exhibiting the same volatility can have very different surprise values.

Following [19], $H^{(1)}$ denotes the entropy rate associated with the state transitions and $H^{(2)}$ is the second order of informativeness. Utilizing theses building blocks, the next expression can be defined as

$$\Delta = \frac{\gamma_K^{[0,T]} - K \cdot H^{(1)}}{\sqrt{K \cdot H^{(2)}}}. \tag{4}$$

This is the surprise of a price trajectory, centered by its expected value, i.e., the entropy rate multiplied by the number of transitions, and divide it by the square root of its variance, i.e., the second order of informativeness multiplied by the number of transitions. It can be shown that

$$\Delta \to \mathcal{N}(0, 1), \text{ for } K, \to \infty, \tag{5}$$

by virtue of the central limit theorem [33]. In other words, for large K, Δ converges to a normal distribution. Equation (4) now allows for the introduction of our probability indicator $\mathcal{L}$, defined as

$$\mathcal{L} = 1 - \Theta\left(\frac{\gamma_K^{[0,T]} - K \cdot H^{(1)}}{\sqrt{K \cdot H^{(2)}}}\right), \tag{6}$$

where Θ is the cumulative distribution function of normal distributions. Thus, an unlikely price trajectory, strongly deviating form a Brownian motion, leads to a large surprise and hence $\mathcal{L} \approx 0$. We can now quantify when markets show normal behavior, where $\mathcal{L} \approx 1$. Again, the reader is referred to [19] for more details.

We now assess how the overshoot event ω should be chosen. The standard framework for coastline trading dictates, that an overshoot event occurs in the price trajectory when the price moves by δ in the overshoots' direction after a directional change. In the context of the probability indicator, we depart from this procedure and define the overshoots to occur when the price moves by $2.525729 \cdot \delta$. This value comes from maximizing the second order informativeness $H^{(2)}$ and guarantees maximal variability of the probability indicator $\mathcal{L}$. For details see [19].

The probability indicator $\mathcal{L}$ can now be used to navigate the trading models through times of severe market stress. In detail, by slowing down the increase of the inventory of agents during price overshoots, the overall trading models exposure experiences smaller drawdowns and better risk-adjusted performance. As a simple example, when an agent cascades, i.e., increases its inventory, the unit size is reduced in times where $\mathcal{L}$ starts to approach zero.

For the trading model, the probability indicator is utilized as follows. The default size for cascading is one unit (lot). If $\mathcal{L}$ is smaller than 0.5, this sizing is reduced to 0.5, and finally if $\mathcal{L}$ is smaller than 0.1, then the size is set to 0.1.

Implementing the above mentioned measures allowed the trading model to safely navigate treacherous terrain, where it derailed in the past. However, there was still one crucial insight missing, before a successful version of the Alpha Engine could be designed. This last insight evolves around a subtle recasting of thresholds which has profound effects on the resulting trading model performance.

3.6 The Final Pieces of the Puzzle

Coming back full circle, the focus was again placed on the nature of the event based formalism. By allowing for new degrees of freedom, the trading model puzzle could be concluded. What before were rigid and static thresholds are now allowed to breathe, giving rise to asymmetric thresholds and fractional position changes.

In the context of directional changes and overshoots, an innocuous question to ask is whether the threshold defining the events should depend on the direction of the current market. In other words, does it make sense to introduce a threshold that is a function of the price move direction? Analytically

$$\delta \rightarrow \begin{cases} \delta_{\text{up}} & \text{for increasing prices;} \\ \delta_{\text{down}} & \text{for decreasing prices.} \end{cases} \tag{7}$$

These asymmetric thresholds now register directional changes at different values of the price curve, depending on the direction of the price movement. As a consequence $\omega = \omega(\delta_{\text{up}}, \delta_{\text{down}})$ denotes the length of the overshoot corresponding to the new upward and downward directional change thresholds. By virtue

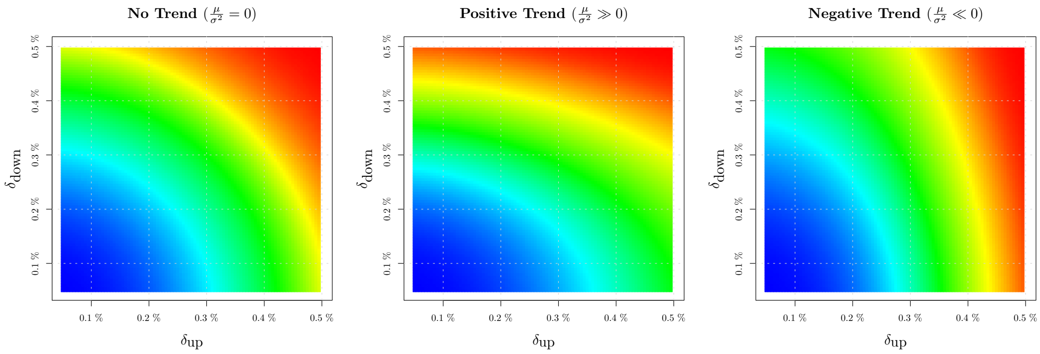

Figure 7: Monte Carlo simulation of the number of directional changes N, seen in Equation (9), as a function of the asymmetric directional change thresholds δ_{up} and δ_{down}, for a Brownian motion, defined by μ and σ. The left-hand panel shows a realization with no trend, while the other two panels have an underlying trend.

of the overshoot size scaling law

$$\langle \omega_{\text{up}} \rangle = \delta_{\text{up}}, \qquad \langle \omega_{\text{down}} \rangle = \delta_{\text{down}}. \tag{8}$$

To illustrate, let P_t be a price curve, modeled as an arithmetic Brownian motion B_t with trend μ and volatility σ, meaning $dP_t = \mu dt + \sigma dB_t$. Now the expected number of upward and downward directional changes during a time interval $[0, T]$ is a function

$$N = N(\delta_{\text{up}}, \delta_{\text{down}}, \mu, \sigma, [0, T]). \tag{9}$$

In Figure 7 the result of a Monte Carlo simulation is shown. For the situation with no trend (left-hand panel) we see the contour lines being perfect circles. In other words, by following any defined circle, the same number of directional changes are found for the corresponding asymmetric thresholds. Details about the analytical expressions and the Monte Carlo simulation regarding the number of directional changes can be found in [20].

This opens up the space of possibilities, as up to now, only the 45-degree line in all panels of Figure 7 were considered, corresponding to symmetric thresholds $\delta = \delta_{\text{up}} = \delta_{\text{down}}$. For trending markets, one can observe a shift in the contour lines, away from the circles. In a nutshell, for a positive trend the expected number of directional changes is larger if $\delta_{\text{up}} > \delta_{\text{down}}$. This reflects the fact that an upward trend is naturally comprised of longer up-move segments. The contrary is true for down moves.

Now it is possible to introduce the notion of invariance as a guiding principle. By rotating the 45-degree line in the correct manner for trending markets, the number of directional changes will stay constant. In other words, if the trend is known, the thresholds can be skewed accordingly to compensate. However, it is not trivial to construct a trend indicator that is predictive and not only reactive.

A workaround is found by taking the inventory as a proxy for the trend. In detail, the expected inventory size I for all agents in normal market conditions can be used to gauge the trend: $\mathbb{E}[I(\delta_{\text{up}}, \delta_{\text{down}})]$ is now a measure of trendiness and hence triggers threshold skewing. In other words, by taking the inventory as an invariant indicator, the 45-degree line can be rotated due to the asymmetric thresholds, counteracting the trend.

A more mathematical justification can be found in the approach of what is known as "indifference prices" in market making. This method can be translated into the context of intrinsic time and agent's inventories. It then mandates that the utility (or preference) of the whole inventory should stay the same for skewed thresholds and inventory changes. In other words, how can the thresholds be changed in a way that "feels" the same as if the inventory increases or decreased by one unit? Expressed as equations

$$U(\delta_{\text{down}}, \delta_{\text{up}}, I) = U(\delta^{*}_{\text{down}}, \delta^{*}_{\text{up}}, I + 1), \tag{10}$$

and

$$U(\delta_{\text{down}}, \delta_{\text{up}}, I) = U(\delta^{**}_{\text{down}}, \delta^{**}_{\text{up}}, I - 1), \tag{11}$$

where U represents a utility function. The thresholds δ^{*}_{up}, δ^{*}_{down}, δ^{**}_{up}, and $\delta^{**}_{\text{down}}$ are "indifference" thresholds.

A pragmatic implementation of such an inventory-driven skewing of thresholds is given by the following equation, corresponding to a long position

$$\frac{\delta_{\text{down}}}{\delta_{\text{up}}} = \begin{cases} 2 & \text{if } I \geq 15; \\ 4 & \text{if } I \geq 30. \end{cases} \tag{12}$$

For a short position, the fractions are inverted

$$\frac{\delta_{\text{up}}}{\delta_{\text{down}}} = \begin{cases} 2 & \text{if } I \leq -15; \\ 4 & \text{if } I \leq -30. \end{cases} \tag{13}$$

In essence, in the presence of trends, the overshoot thresholds decrease as a result of the asymmetric directional change thresholds.

This also motivates a final relaxation of a constraint. The final ingredient of the Alpha Engine are fractional position changes. Recall that coastline trading is simply an increase or decrease in the position size at intrinsic time events. This cascading and de-cascading was done by one unit. For instance, increasing a short position size by one unit if the price increases and reaches an upward overshoot. To make this procedure compatible with asymmetric thresholds, the new cascading and de-cascading events, resulting from the asymmetric threshold, are now done with a fraction of the original unit. The fractions are also dictated by Equations (12) and (13). In effect, the introduction of asymmetric thresholds leads to a subdivision of the original threshold into smaller parts, where the position size is changed by sub-units on these emerging threshold.

An example is shown in Figure 8. Assuming that a short position was opened at a lower price than the minimal price in the illustration, the directional change

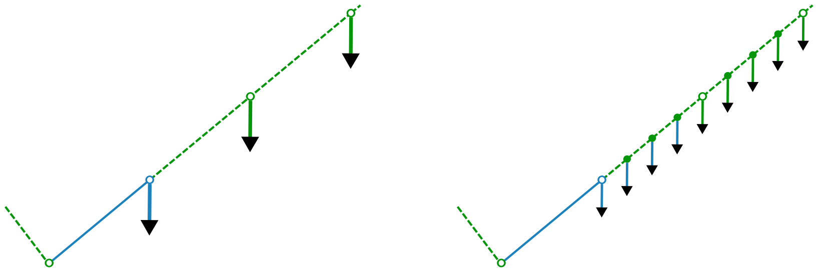

Figure 8: Cascading with asymmetric thresholds. A stylized price curve is shown in both panels. (*Left*) The original (symmetric) setup with an upward directional change event (continuous line) and two overshoots (dashed lines). Short position size increments are shown as downward arrows. (*Right*) The situation corresponding to asymmetric thresholds, where intrinsic time accelerates and smaller position size increments are utilized for coastline trading. See details in text.

will trigger a cascading event. In other words, one (negative) unit of exposure (symbolized by the large arrows) is added to the existing short position. The two overshoot events in the left-hand panel trigger identical cascading events. In the right-hand panel, the same events are augmented by asymmetric thresholds. Now $\omega_{\text{up}} = \omega_{\text{down}}/4$. As a result, each overshoot length is divided into four segments. The new cascading regime is as follows: increase the position by one-fourth of a (negative) unit (small arrow) at the directional change and another fourth at the first, second, and third asymmetric overshoots each. In effect, the cascading event is "smeared out" and happens in smaller unit sizes over a longer period. For the cascading events at the first and second original overshoot, this procedure is repeated.

This concludes the final chapter in the long history of the trading model development. Many insights from diverse fields were consolidated and a unified modelling framework emerged.

4 The Nuts and Bolts: A Summary of the Alpha Engine

All the insights gained along this long journey need to be encapsulated and translated into algorithmic concepts. In this section, we summarize in detail the trading model behavior and the specify the parameters.

The intrinsic time scale dissects the price curve into directional changes and overshoots, yielding an empirical scaling law equating the length of the overshoot ω to the size of the directional change threshold δ, i.e., $\langle\omega\rangle \approx \delta$. This scaling law creates an event based language, that clocks the trading model. In essence, intrinsic time events define the coastline trading behavior with its hallmark cascading and de-cascading events. In other words, the discrete price curve with occurrences of intrinsic time events triggers an increase or decrease in position sizes.

In detail, an intrinsic event is either a directional change or a move of size δ in the direction of the overshoot. For each exchange rate, we assign four coastline traders $\text{CT}_i[\delta_{\text{up/down}}(i)],\ i = 1, 2, 3, 4$, that operate at various scales, with upward and downward directional change thresholds equaling $\delta_{\text{up/down}}(1) = 0.25\%, \delta_{\text{up/down}}(2) = 0.5\%, \delta_{\text{up/down}}(3) = 1.0\%$, and $\delta_{\text{up/down}}(4) = 1.5\%$.

The default size for cascading and de-cascading a position is one unit (lot). The probability indicator $\mathcal{L}_i$, assigned to each coastline trader, is evaluated on the fixed scale $\delta(i) = \delta_{\text{up/down}}(i)$. As a result, its states are directional changes of size $\delta(i)$ or overshoot moves of size $2.525729 \cdot \delta_i$. The default unit size for cascading is reduced to 0.5 if $\mathcal{L}_i$ is smaller than 0.5. Additionally, if $\mathcal{L}_i$ is smaller than 0.1, then the size is further reduced to 0.1.

In case a coastline trader accumulates an inventory with a long position greater than 15 units, the upward directional change threshold $\delta_{\text{up}}(i)$ is increased to 1.5 of its original size, while the downward directional change threshold $\delta_{\text{down}}(i)$ is decreased to 0.75 of its original size. In effect, the ratio for the skewed thresholds is $\delta_{\text{up}}(i)/\delta_{\text{down}}(i) = 2$. The agent with the skewed thresholds will cascade when the overshoot reaches 0.5 of the skewed threshold, i.e., half of the original threshold size. In case the inventory with long position is greater than 30, then the upward directional change threshold $\delta_{\text{up}}(i)$ is increased to 2.0 of its original size and the downward directional change threshold $\delta_{\text{down}}(i)$ is decreased to 0.5. The ratio of the skewed thresholds now equals $\delta_{\text{up}}(i)/\delta_{\text{down}}(i) = 4$. The agent with these skewed thresholds will cascade when the overshoot extends by 0.25 of the original threshold, with one-fourth of the specified unit size. This was illustrated in the right-hand panel of Figure 8. The changes in threshold lengths and sizing is analogous for short inventories.

This concludes the description of the trading model algorithm and the motivation of the chosen modeling framework. Recall that the interested reader can download the code from GitHub [35].

5 Conclusion and Outlook

The trading model algorithm described here is the result of a meandering journey that lasted for decades. Guided by an overarching event-based framework, recasting time as discrete and driven by activity, elements from complexity theory and information theory were added. In a nutshell, the proposed trading model is defined by a set of simple rules executed at specific events in the market. This approach to designing automated trading models yields an algorithm that fulfills many desired features. Its parsimonious, modular, and self-similar design results in behaviour that is profitable, robust, and adaptive.

Another crucial feature of the trading model is that it is designed to be counter trend. The coastline trading ensures that positions, which are going against a trend, are maintained or increased. In this sense, the models provide liquidity to the market. When market participants want to sell, the investment strategy will buy and vice versa. This market-stabilizing feature of the model is beneficial to the markets as a whole. The more such strategies are implemented, the less we expect to see runaway markets but healthier market conditions overall. By construction, the trading model only ceases to perform in low-volatility markets.

It should be noted that the model framework presented here can be realized with reasonable computational resources. The basic agent-based algorithm shows profitable behavior for four directional change thresholds, on which the positions (agents) live. However, by adding more thresholds the model behavior is expected to become more robust, as more information coming from the market can be processed by the trading model. In other words, by increasing the model complexity the need for performant computing becomes relevant for efficient prototyping and back-testing. In this sense, we expect the advancements in high performance computing in finance to positively impact the Alpha Engine's evolution.

Nevertheless, with all the merits of the trading algorithm presented here, we are only at the beginning. The Alpha Engine should be understood as a prototype. It is, so to speak, a proof of concept. For one, the parameter space can be explored in greater detail. Then, the model can be improved by calibrating the various exchange rates by volatility, or by excluding illiquid ones. Furthermore, the model treats all the currency pairs in isolation. There should be a large window of opportunity for increasing the performance of the trading model by introducing correlation across currency pairs. This is a unique and invaluable source of information not yet exploited. Finally, a whole layer of risk management can be implemented on top of the models.

We hope to have presented a convincing set of tools motivated by a consistent philosophy. If so, we invite the reader to take what is outlined here and improve upon it.

A A History of Ideas

This section is a personal recount of the historical events that would ultimately lead to the development of the trading model algorithm outlined in this chapter, told by Richard B. Olsen:

The development of the trading algorithm and model framework dates back to my studies in the mid 70s and 80s. From the very start my interests in economics were influenced by my admiration of the scientific rigor of natural sciences and their successful implementations in the real world. I argued that the resilience of the economic and political systems depends on the underlying economic and political models. Motivated to contribute to the well being of society I wanted to work on enhancing economic theory and work on applying the models.

I first studied law at the University of Zurich and then, in 1979, moved to Oxford to study philosophy, politics, and economics. In 1980 I attended a course on growth models by James Mirrlees, who, in 1996, would receive a Nobel prize in economics. In his first lecture he discussed the short-comings of the models, such as [25]. He explained that the models are successful in explaining growth as long as there are no large exogenous shocks. But unanticipated events are inherent to our lives and the economy at large. I thus started to search for a model framework that can both explain growth and handle unexpected exogenous shocks. I spent one year studying the Encyclopedia Britannica and found my inspiration in relativity theory.

In my 1981 PhD thesis, titled "Interaction between Law and Society", at the University of Zurich, I developed a new model framework that describes in an abstract language, how interactions in the economy occur. At the core of the new approach are the concepts of object, system, environment, and event-based intrinsic time. Every object has its system that comprises all the forces that impact and influence the object. Outside the system is its environment with all the forces that do not impact the object. Every object and system has its own frame of reference with an event-based intrinsic time scale. Events are interactions between different objects and their systems. I concluded that there is no abstract and universal time scale applicable to every object. This motivated me to think about the nature of time and how we use time in our everyday economic models.

After finishing my studies, I joined a bank working first in the legal department, then in the research group, and finally joined the foreign exchange trading desk. My goal was to combine empirical work with academic research, but was disappointed with the pace of research at the bank. In the mid 80s, there was the first buzz about start-ups in the United States. I came up with a business idea: banks have a need for quality information to increase profitability, so there should be a market for quality real time information.

I launched a start-up with the name of Olsen & Associates. The goal was to build an information system for financial markets with real time forecasts and trading recommendations using tick-by-tick market data. The product idea combined my research interest with an information service, which would both improve the quality of decision-making in financial markets and generate revenue to fund further research. The collection of tick market data began in January 1986 from Reuters. We faced many business and technical obstacles, where data storage cost was just one of the many issues. After many setbacks we successfully launched our information service and eventually acquired 60 big to mid-sized banks across Europe as customers.

In 1990, we published our first scientific paper [29] revealing the first scaling law. The study showed that intraday prices have the same scaling law exponent as longer-term price movements. We had expected two different exponents: one for intraday price movements, where technical factors dictate price discovery, and another for longer-term price movements that are influenced by fundamentals. The result took us by surprise and was evidence that there are universal laws that dictate price discovery at all scales. In 1995 we organized the first high frequency data conference in Zurich, where we made a large sample of tick data available to the academic community. The conference was a big success and boosted market microstructure research, which was in its infancy at that time. In the following years we conducted exhaustive research testing all possible model approaches to build a reliable forecasting service and trading models. Our research work is described in the book [15]. The book covers data collection and filtering, basic stylized facts of financial market time series, the modelling of 24 hour seasonal volatility, realized volatility dynamics, volatility processes, forecasting return and risks, correlation, and trading models. For many years the book was a standard text for major hedge funds. The actual performance of our forecasting and trading models was, however, spurious and disappointing. Our models were best in class, but we had not achieve a breakthrough.

Back in 1995, we were selling tick-by-tick market data to top banks and created a spinoff under the name of OANDA to market a currency converter on the emergent Internet and eventually build a foreign exchange market making business. The OANDA currency converter was an instant success. At the start of 2001, we were completing the first release of our trading platform. At the same time, Olsen & Associates was a treasure store of information and risk services, but did not have cash to market the products and was struggling for funding. When the Internet bubble burst and markets froze, we could not pay our bills and the company went into default. I was able to organize a bailout with a new investor. He helped to salvage the core of Olsen & Associates with the aim of building a hedge fund under the name of Olsen Ltd and buying up the OANDA shares.

In 2001, the OANDA trading platform was a novelty in the financial industry: straight through processing, one price for everyone, and second-by-second interest payments. At the time, these were true firsts. At OANDA, a trader could buy literally 1 EUR against USD at the same low spread as a buyer of 1 million EUR against USD. The business was an instant success. Moreover, the OANDA trading platform was a research laboratory to analyse the trades of ten thousands of traders, all buying and selling at the same terms and conditions, and observe their behaviour patterns in different market environments. I learned hands on, how financial markets really work and discovered that basic assumptions of market efficiency that we had taken for granted at Olsen & Associates were inappropriate. I was determined to make a fresh start in model development.

At Olsen Ltd I made a strategic decision to focus exclusively on trading model research. Trading models have a big advantage over forecasting models: the profit and losses of a trading model are an unambiguous success criterion of the quality of a model. We started with the forensics of the old model algorithms and discovered that the success and failure of a model depends critically on the definition of time and how data is sampled. Already at Olsen & Associates we were sensitive to the issue of how to define time and had rescaled price data to account for the 24 hour seasonality of volatility, but did not succeed with a more sophisticated rescaling of time. There was one operator that we had failed to explore. We had developed a directional change indicator and had observed that the indicator follows a scaling law behaviour similar to the absolute price change scaling law [21]. This scaling law was somehow forgotten and was not mentioned in our book [15]. I had incidental evidence that this operator would be successful to redefine time, because traders use such an operator to analyse markets. The so-called point and figure chart replaces the x-axis of physical time with an event scale. As long as a market price moves up, the prize stays frozen in the same column. When the price moves down by a threshold bigger than the box size, the plot moves to the next column. A new column is started, when the price reverses its direction.

Then I also had another key insight of the path dependence of market prices from watching OANDA traders. There was empirical evidence that a margin call of one trader from anywhere in the world could trigger a whole cascade of margin calls in the global foreign exchange markets in periods of herding behaviour. Cascades of margin calls wipe out whole cohorts of traders and tilt the market composition of buyers and sellers and skew the long-term price trajectory. Traditional time series models cannot adequately model these phenomena. We decided to move to agent-based models to better incorporate the emergent market dynamics and use the scaling laws as a framework to calibrate the algorithmic behaviour of the agents. This seemed attractive, because we could configure self-similar agents at different scales.

I was adamant to build bare-bone agents and not to clutter our model algorithms with tools of spurious quality. In 2008, we were rewarded with major breakthrough: we discovered a large set of scaling laws [16]. I expected that model development would be plain sailing from thereon. I was wrong. The road of discovery was much longer than anticipated. Our hedge fund had several significant drawdowns that forced me to close the fund in 2013. At OANDA, things had also deteriorated. After raising 100 million USD for 20% of the company in 2007, I had become chairman without executive powers. OANDA turned into a conservative company and lost its competitive edge. In 2012, I left the board.

In July 2015, I raised the first seed round for Lykke, a new startup. Lykke builds a global marketplace for all asset classes and instruments on the blockchain. The marketplace is open source and a public utility. We will earn money by providing liquidity with our funds and or customer's funds, with algorithms as described in this paper.

B Supplementary Material

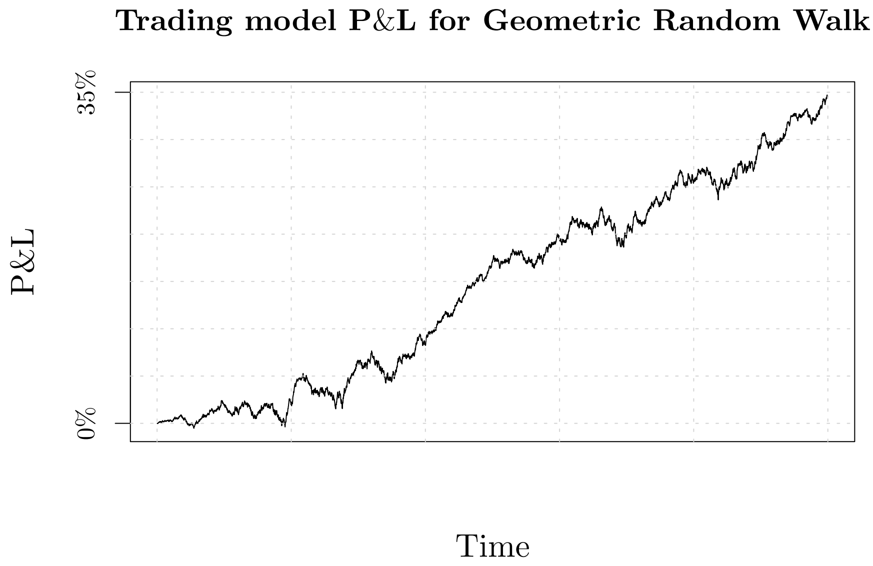

Figure 9: Profit & Loss for a time series, generated by a geometric random walk of 10 million ticks with annualized volatility of 25%. The average of 60 Monte Carlo simulations is shown. In the limiting case, the P&L curve becomes a smooth increasing line.

%	Jan	Feb	Mar	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec	Year
2006	0.16	0.15	0.07	0.12	0.22	0.17	0.19	0.20	0.18	0.08	-0.00	0.04	**1.58**
2007	0.08	0.22	0.14	0.02	-0.05	-0.03	0.32	0.59	0.07	0.11	0.47	0.20	**2.03**
2008	0.24	0.07	0.05	0.50	0.26	0.09	0.26	0.16	0.66	2.22	1.27	0.98	**6.03**
2009	1.14	1.41	1.17	1.00	0.75	0.59	0.22	0.19	-0.13	0.28	0.06	0.25	**7.70**
2010	0.15	-0.34	0.24	0.14	0.30	0.17	0.27	-0.02	0.03	0.06	0.14	-0.31	**1.42**
2011	0.45	0.13	0.11	-0.16	0.04	-0.06	-0.40	0.43	0.45	-0.03	0.32	-0.03	**0.97**
2012	-0.08	0.19	0.29	0.08	-0.12	0.15	-0.20	0.23	0.10	0.13	0.12	0.11	**0.86**
2013	-0.17	-0.01	-0.10	-0.08	0.32	0.52	0.04	0.24	-0.10	0.01	-0.01	-0.16	**0.77**

Table 1: Monthly performance of the unleveraged trading model. The P&L is given in percentages. All 23 currency pairs are aggregated.

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