# Deep Hedging

### Machine-driven trading of derivatives under market frictions

Swissquote Conference 2018 on Machine Learning in Finance Geneva, Nov 9<sup>th</sup> 2018

Dr. Hans Buehler

J. P. Morgan

Joint work with
Lucas Gonon (ETH), Jonathan Kochems (JPM), Baranidharan Mohan(JPM), Josef Teichmann
(ETH), Hans Buehler (JPM)

#### Disclaimer

This material is not the product of J.P. Morgan's Research Department. It is not a research report and is not intended as such. This material is provided for informational purposes only and is subject to change without notice. It is not intended as research, a recommendation, advice, offer or solicitation to buy or sell any financial product or service, or to be used in any way for evaluating the merits of participating in any transaction. Please consult your own advisors regarding legal, tax, accounting or any other aspects including suitability implications for your particular circumstances. J.P. Morgan disclaims any responsibility or liability whatsoever for the quality, accuracy or completeness of the information herein, and for any reliance on, or use of this material in any way. This material is provided on a confidential basis and may not be reproduced, redistributed or transmitted, in whole or in part, without the prior written consent of J.P. Morgan. Any unauthorized use is strictly prohibited. The products and/or services mentioned herein may not be suitable for your particular circumstances and may not be available in all jurisdictions or to all clients. Clients should contact their salespersons at, and execute transactions through, a J.P. Morgan entity appropriately licensed in the client's home jurisdiction unless governing law permits otherwise. This material is a "solicitation" of derivatives business only as that term is used within CFTC Rule 1.71 and 23.605. Where this material is an "investment recommendation" as that term is defined in MAR visit: www.jpmm.com/#mardisclosures. This material is subject to terms at: www.jpmorgan.com/salesandtradingdisclaimer.

© 2017 JPMorgan Chase & Co. All rights reserved. J.P. Morgan is a marketing name for investment banking businesses of J.P. Morgan Chase & Co. and its subsidiaries and affiliates worldwide. Bank products and services, including certain lending, derivative and other commercial banking activities, are offered by JPMorgan Chase Bank N.A. (JPMCB), including through its authorized branches and other global affiliates registered with local authorities as appropriate. Securities products and services, including execution services, are offered in the United States by J.P. Morgan Securities LLC (JPMS LLC), in EMEA by J.P. Morgan Securities plc (JPMS plc) where permitted and in other jurisdictions worldwide by other appropriately licensed global affiliates. JPMCB, JPMS LLC and JPMS plc are principal subsidiaries of JPMorgan Chase & Co. For information on which legal entities offer investment banking products and services in each jurisdiction, please consult: <a href="https://www.jpmorgan.com/ib-legal-entities">www.jpmorgan.com/ib-legal-entities</a>. For important disclosures in respect of securities transactions, please consult: <a href="https://www.jpmorgan.com/securities-transactions">www.jpmorgan.com/securities-transactions</a> and in respect of over-the-counter equity derivatives transactions, please consult: <a href="https://www.jpmorgan.com/securities-transactions">www.jpmorgan.com/securities-transactions</a>.

For additional regulatory disclosures, please consult: www.jpmorgan.com/disclosures.

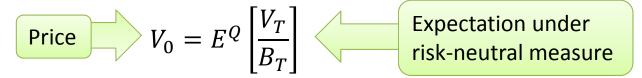
### Outline

- Models in an exotic derivatives business
- Teaching a machine to think like a trader
- First steps: toy model trading
- Moving further into the real world

#### How are models used in an exotic derivatives business?

#### **Pricing new trades**

Classical risk-neutral models are ubiquitous



- Disregard any existing portfolio and price the derivative under the assumption that perfect replication is possible
- Apply local adjustments: hedging costs (trader's estimate), model limitation adjustments, ...
- For larger trades, consider global adjustments depending on existing portfolio: credit charge, concentration charge, etc.

#### How are models used in an exotic derivatives business?

#### Hedging

Compute the price with the usual classical model

$$V_0 = E^Q \left[ \frac{V_T}{B_T} \right]$$

Then compute "greeks"

$$\frac{\partial V_0}{\partial X}$$

- For factors which are stochastic in the model, and parameters which aren't (e.g. interest rates in a local volatility model)
- Based on the greeks, decide which hedging instruments to buy/sell
  - The right hedge is not just the model risk
  - Traders adjust the actual traded risk with "experience/skill"
  - He/she needs to be aware of transaction costs, market dynamics (such as vol-spot correlation), concentration and liquidity risk...

#### How are models used in an exotic derivatives business?

#### **Apply constraints**

- Internal: control the risks we take, ensure efficient use of capital
- External: regulatory, legal

#### **Examples:**

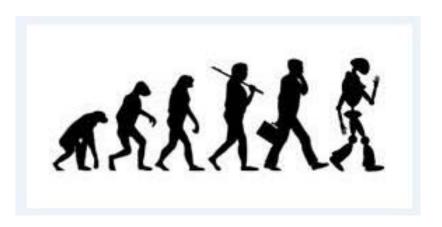
Direct risk and stress limits based on the model:

$$L < \frac{\partial V_0}{\partial X} < U$$

$$V_0(X) - V_0(X + \text{stress}) < M$$

- Limits on CVaR
- Capital requirements many determining factors
- Short selling bans
- These constraints are not usually part of the valuation model

### Beyond the classical approach



- We want to increase automation in the business
- The risk management model needs to do more

- It should include transaction costs, lack of liquidity, and constraints
- This means accepting that perfect replication is impossible...

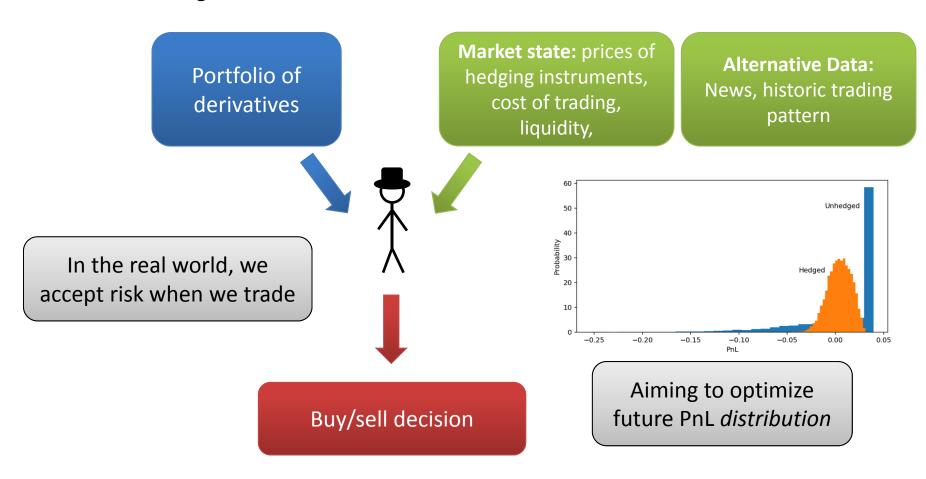


### Outline

- Models in an exotic derivatives business
- Teaching a machine to think like a trader
- First steps: toy model trading
- Moving further into the real world

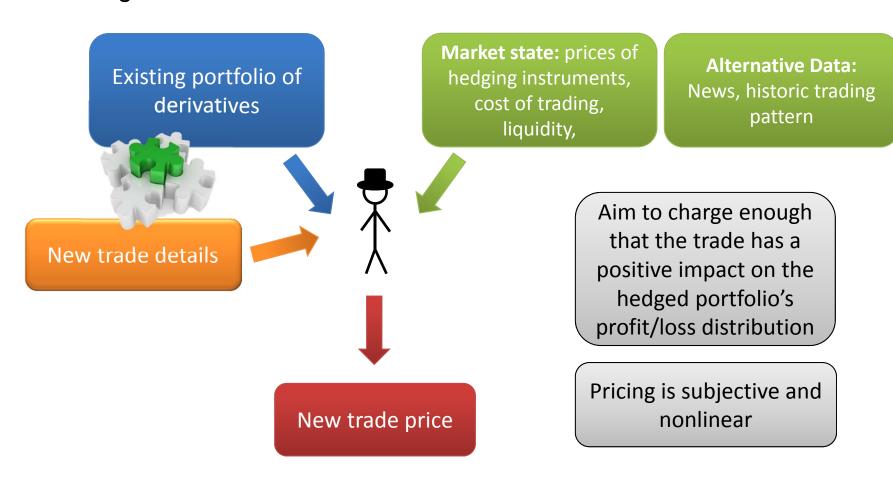
### Trading inputs and outputs

Risk management



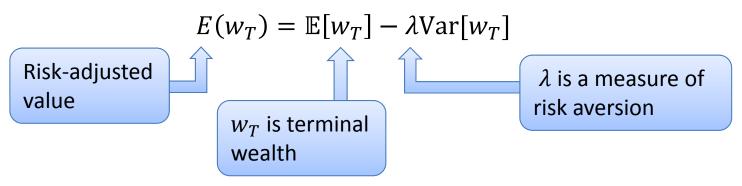
### Trading inputs and outputs

Pricing



### How to compare profit/loss distributions?

- We could use classical "Markoviz" portfolio optimization
  - Maximize expected return while penalizing variance



- Note that  $E(w_T)$  is a function on the *distribution* of terminal wealth
- But mean-variance is not a good measure if terminal wealth is not normally distributed
  - Exist on-monotone cases where  $X \ge Y$  but E(X) < E(Y)

## How to compare profit/loss distributions?

- What are sensible conditions for our risk-adjusted value function  $E(w_T)$ ?
- Monotonicity

$$X \ge Y \Rightarrow E(X) \ge E(Y)$$

More is better

Convexity

$$E(\alpha X + (1 - \alpha)Y) \ge \alpha E(X) + (1 - \alpha)E(Y), \alpha \in [0,1]$$

We are risk-averse

Cash invariance

$$E(X+c) = E(X) + c$$

There is no risk adjustment for cash

 $-E(\cdot)$  is a **convex risk** measure

## How to compare profit/loss distributions?

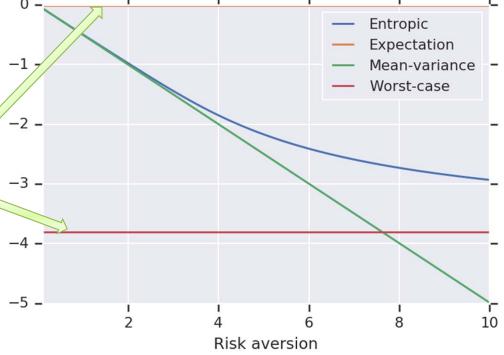
- We will mostly use the *entropic* measure:  $E(X) = -\frac{1}{\lambda} \ln \mathbb{E}[e^{-\lambda X}]$
- Equivalent to mean-variance for small risk-aversion parameter  $\lambda$ :

$$-\frac{1}{\lambda} \ln \mathbb{E}[e^{-\lambda X}] = \mathbb{E}[X] - \frac{1}{2} \lambda \operatorname{Var}[X] + \cdots$$

• Example:  $X \sim \mathcal{N}(0,1)$ 

Plot risk-adjusted value

Bounded by the two extreme risk-adjusted values: risk-neutral and worst-case



### Hedging

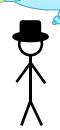
- We can now express preferences on future profit/loss distributions
- Hedging is the act of buying and selling "hedging instruments" to optimize that distribution

 $a_t^{\pi} = a_t^{\pi}(t, s_t)$ 

A hedging strategy is a function:

Liquid instruments with observable prices

 $\pi$  parameterized the strategy; e.g. as the weights of neural networks



State  $s_t$  is the history of everything including our current book

- It tells us how much of each hedging instrument to buy or sell at each time t, for every possible state  $s_t$
- Not all actions are possible in general  $a_t^{\pi}$  will be subject to limits which are also state-dependent (e.g. short-sell constraints)

## Hedging

• How does the hedging strategy  $\pi$  contribute to the terminal profit/loss?

$$w_T(\pi; z) = \sum_{j=1}^T \delta_j^{\pi} \cdot h_j + z_j - a_j^{\pi} \cdot H_j - c_j^{\pi}$$

 $\delta_j^\pi$ : accumulated current positions ("deltas"),  $\delta_i^\pi = \delta_{i-1}^\pi + a_i^\pi$ 

 $h_j$ : cashflows generated by hedging instruments

Note: all cash flows are discounted

 $c_j^{\pi}$ : transaction costs incurred,  $c_i^{\pi} = c(a_i^{\pi}, s_i)$ 

 $H_j$ : mid prices of hedging instruments

 $z_j$ : cashflows from our exotic derivatives portfolio

### Hedging

- The terminal profit/loss is not deterministic our task is to optimize it
- That means maximizing

$$E[w_T(\pi; z)] = E\left[\sum_{j=1}^T \delta_j^{\pi} \cdot h_j + z_j - a_j^{\pi} \cdot H_j - c_j^{\pi}\right]$$

We apply the value function to the distribution over future real-world states

- Two key challenges:
  - How to generate the distribution
  - How to optimize the hedging function

We need to find the optimal function  $a_j^{\pi}$  that meets our constraints.

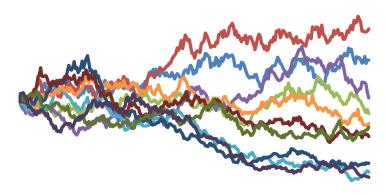
#### Path dependency

The feasible set of allowed actions  $a_j^{\pi}$  depends on past decisions  $a_{j-1}^{\pi}$  ...  $a_0^{\pi}$ .

#### Market simulators

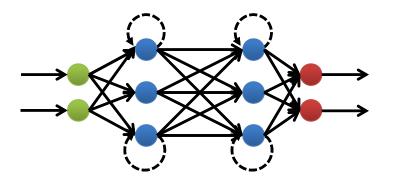
- To generate the profit/loss distribution for a given strategy, we need to simulate future states of the world
  - Prices of available hedging instruments
  - Corresponding cash flows from exotic derivatives
- We should be simulating in the real-world measure, not Q
  - The real world has "statistical arbitrage", i.e. with normal risk aversion some trades statistically make money (e.g. shorting options, sell long-dated bonds).
  - Deep Hedging will attempt to take advantage of these opportunities.
    - Absence of arbitrage =/=> absence of statistical arbitrage (e.g. GBM with drift)
    - Existence of arbitrage =/=> existence of statistical arbitrage (e.g. if risk-aversion is very high)
- For the experiments presented here, will use classical 

  models



### Optimizing the hedging strategy

• We use a *deep neural network* to represent the strategy  $a_i^{\pi}$ 



Inputs:

Prices of all hedging instruments

- Current market state
- Relevant product state variables

Harvested automatically

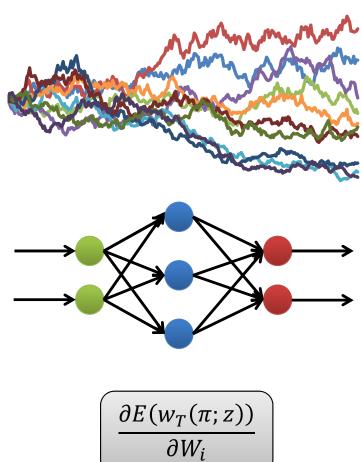
- LSTM cells to capture path dependence
  - Potentially important when we have transaction costs
  - Allows memory of our previous hedging decisions

### **Deep Hedging**

Simulate many future states of the world

Compute risk-adjusted value on a batch of paths for neural network strategy

> Update network parameters by following gradient



$$\frac{\partial E(w_T(\pi;z))}{\partial W_i}$$

### Outline

- Models in an exotic derivatives business
- Teaching a machine to think like a trader
- First steps: toy model trading
- Moving further into the real world

#### Toy model trading

### Start simple

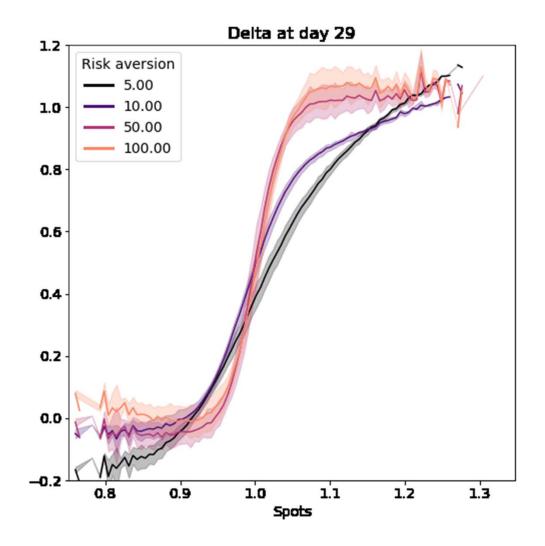
Hedge a short at-the-money 30-day European call

$$z_T = -(S_T - K)^+$$

- Generate paths in Black-Scholes
- Check the impact of transaction costs, risk aversion, and risk limits

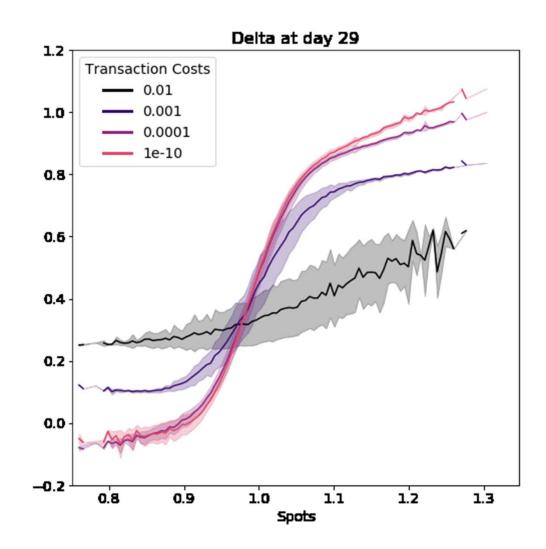
### Risk Aversion (Entropy)

- Vanilla option delta
- 10bps cost
- No limits
- Entropic value
- Black-Scholes simulator



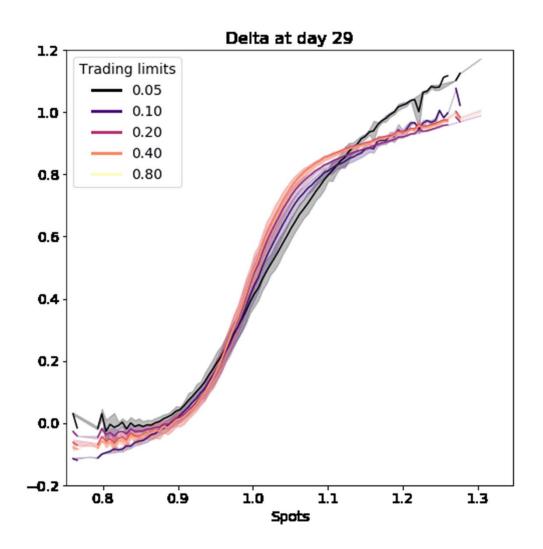
### Transaction costs (Entropy)

- Vanilla option delta
- No limits
- Entropic value
- Risk aversion10
- Black-Scholes simulator



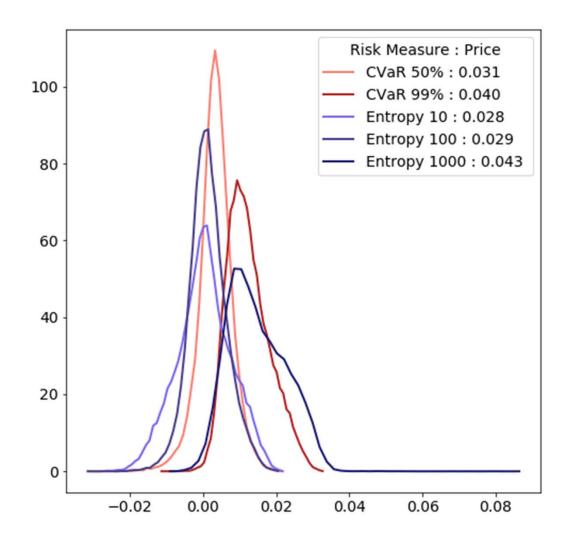
### **Trading limits**

- Vanilla option delta
- 0.01%proportionalcost
- Entropic value
- Risk aversion10
- Black-Scholes simulator



#### Risk measure

- Vanilla optionPnLdistribution
- 0.01% cost
- No limits
- Black-Scholes simulator



### Forward-starting options

- Increase the complexity: simulate with Heston model
- Compute optimal spot-only hedges for forward-starting options

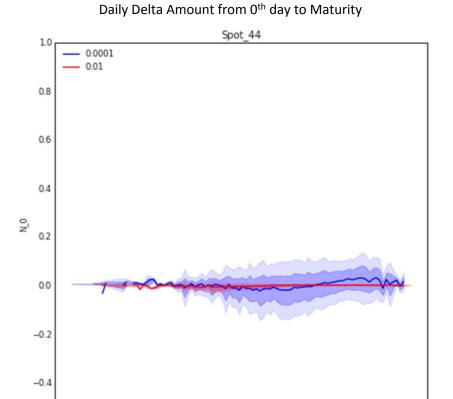
$$z_T = \max\left(0, \frac{S_T}{S_t} - K\right)$$

- 15-day forward start
- 45-day maturity
- Daily hedging
- Entropic value with risk aversion 50
- No limits

### Forward-starting options

Impact of transaction costs on incremental and total delta

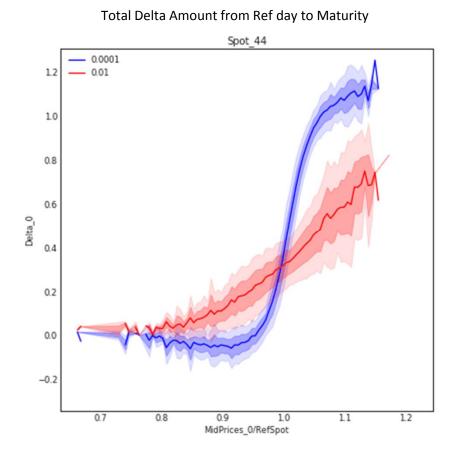
1.1



0.9

MidPrices\_0

1.0



0.8

### Outline

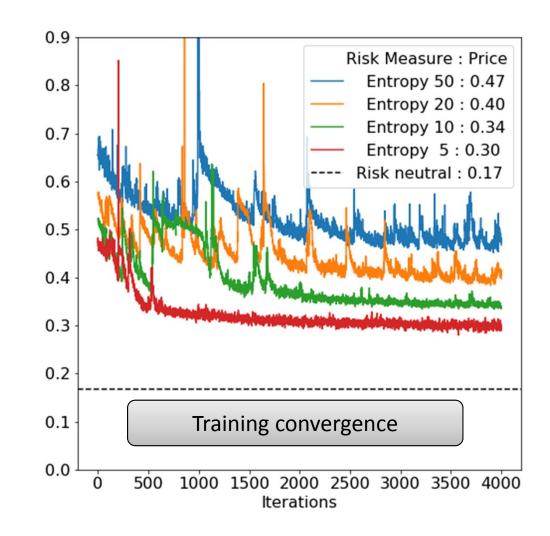
- Models in an exotic derivatives business
- Teaching a machine to think like a trader
- First steps: toy model trading
- Moving further into the real world

#### "Autocallable" note

- Popular retail payoff:
  - Client is short a down-and-in put paid at maturity
  - Upper knockout barrier
  - Fixed coupons until KO
- 0.1% transaction costs
- No limits
- Risk aversion 20
- Entropic value
- Monthly hedging

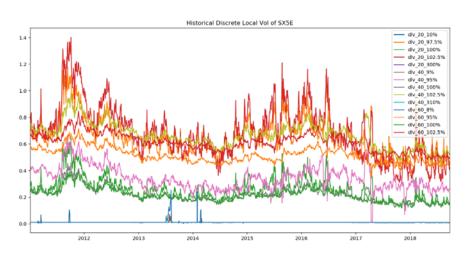
#### Portfolio of autocallables

- Based on a real portfolio
- 0.1% transaction costs
- No limits
- Local volatility simulator
- Monthly hedging



#### Market simulator

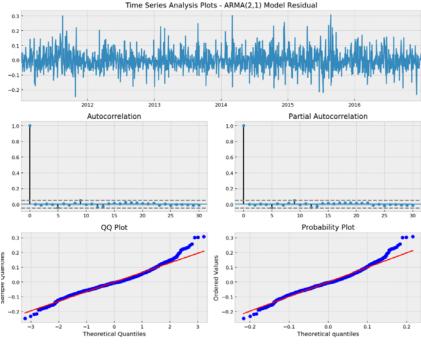
Go beyond "classical" models – build a statistical model instead



Challenge: avoid arbitrage when simulating options

Simulate discrete local volatilities to avoid static arbitrage

Dynamic arbitrage still a challenge



#### Market simulator

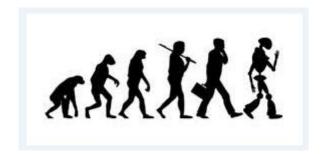
- How do we build a full statistical market simulator that reflects realworld drifts but is arbitrage-free?
  - Does it need to be fully arbitrage-free?
- What about rare events?
  - A statistical simulator is not likely to capture these well
- In particular, we want the model to behave well in a stress scenario, and to price in the risk appropriately
  - Should we insert stress events into the market simulator?
  - With what probability? Historical likelihood?
- For equities we focused on spot and volatility there's lots more
  - Rates, spreads, FX, ...

#### Conclusions

- We formalized the task of pricing and managing the risk of an exotic derivatives portfolio
- Obtaining the optimal hedging strategy is a difficult problem
- Representing the strategy as a neural network makes it tractable
  - Optimization typically takes minutes on CPU for the toy examples here
- So far it works for:
  - Vanillas, cliquets, barrier options, large portfolios
  - With transaction costs and risk limits
  - Simulators based on classical pricing models (Black Scholes, local volatility, Heston)

### Many more interesting challenges ahead

- Developing statistical (P) market simulators for options
- Do we need to compute hedging strategies all the way to maturity?
  - Can we come up with an efficient way to represent a portfolio of exotics as a state?
- How do we choose our risk measure? Can we derive effective realworld risk-measures from the choices people make?
- Go beyond equities: FX, rates, etc.
- Ultimate goal: automated pricing and hedging of exotic derivatives



#### References and thanks

- Credits:
  - Hans Buehler, Lukas Gonon, Josef Teichmann, Hans Buehler, Jonathan Kochems, Barani Mohan, Blanka Horvath, Len Bai, Pradeepta Das
- Paper:
  - Deep Hedging, Hans Buehler, Lukas Gonon, Josef Teichmann, Hans Buehler, <a href="https://arxiv.org/abs/1802.03042">https://arxiv.org/abs/1802.03042</a>
  - Note that the architecture has moved on since the paper