Exit Strategies for COVID-19: An Application of the K-SEIR Model

v1.1 - April 16, 2020

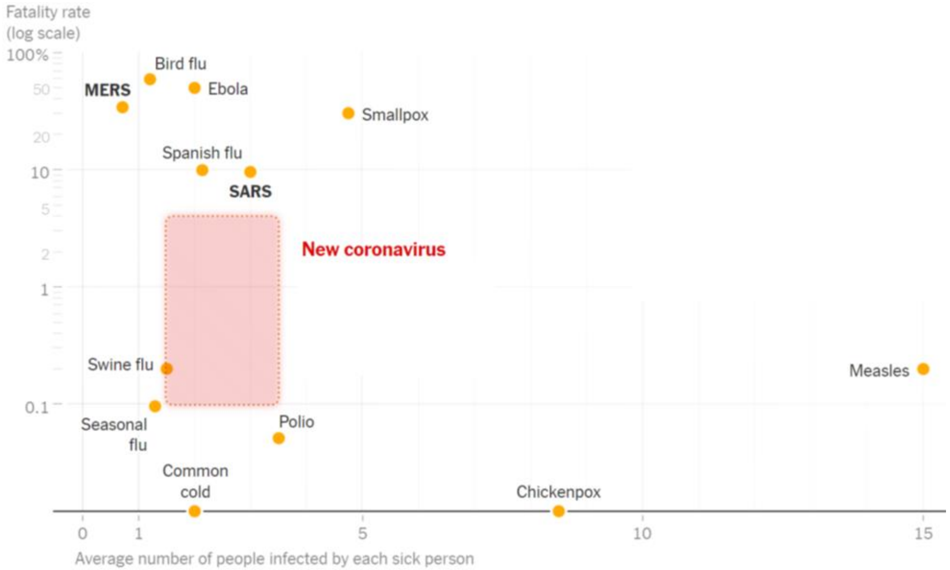
Prof. Alexander Lipton
Hebrew University of Jerusalem
Connection Science, MIT

Prof. Marcos López de Prado
School of Engineering
Cornell University

Background

What is COVID-19?

- COVID-19 is the infectious disease caused by the severe acute respiratory syndrome novel coronavirus (SARS-CoV-2)
- SARS-CoV-2 was first isolated and named on February 11, 2020
- The first cases appeared in
  - Huabei (China), in December 2019
  - New York (U.S), on [January 20, 2020]
- As of April 16, 2020, more than 2.17 million cases have been reported across 210 countries

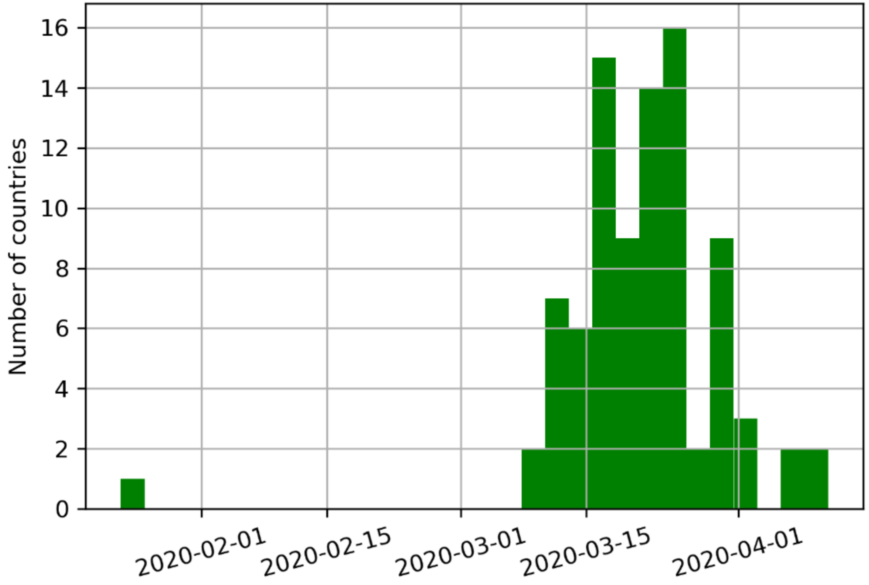


Source: [The New York Times]

Transmissibility vs. fatality rates of various diseases. At this point, these variables are unknown for COVID-19.

Government Responses to COVID-19

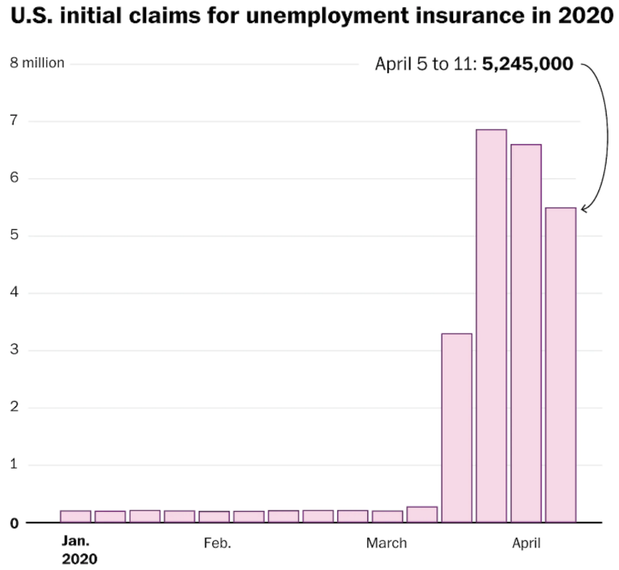
- On March 11, the World Health Organization declared COVID-19 a pandemic
- Numerous countries have instituted strict controls designed at curving the spread of the disease
  - **Social distancing** for the general population
  - **Quarantine** (14 days) of individuals returning from infected areas
  - **Lockdown / shelter-in-place / stay-at-home** of areas with widespread infections
  - **Self-isolation** of individuals who have tested positive or are suspected to be infected



Eighty-eight countries have instituted controls, covering a combined 6.17 billion inhabitants (approx. 79.09% of the World's population). The median start date was March 22, 2020. California was the first State in the U.S. to impose a stay-at-home order, on March 19, 2020.

The Great Shutdown

- COVID-19 is a threat to lives, first and foremost, but also a threat to livelihoods
- While the virus does not necessarily have a meaningful impact on economic output, government interventions have driven many economies to a halt
  - In a matter of weeks, the unemployment rate in the United States went from around 3.5% to approximately 13.5%, the worst since the Great Depression
- Unless lockdowns are resolved, economists expect a drop in U.S. GDP between 8% and 13%
  - A loss between $1.72 trillion and $2.79 trillion in the U.S.
  - During the Great Recession, U.S. GDP fell only 4.3%



Source: The Washington Post

As of April 16, 2020, the U.S. had 22 million unemployed. Job losses over the past 4 weeks erased all jobs created over the past decade.

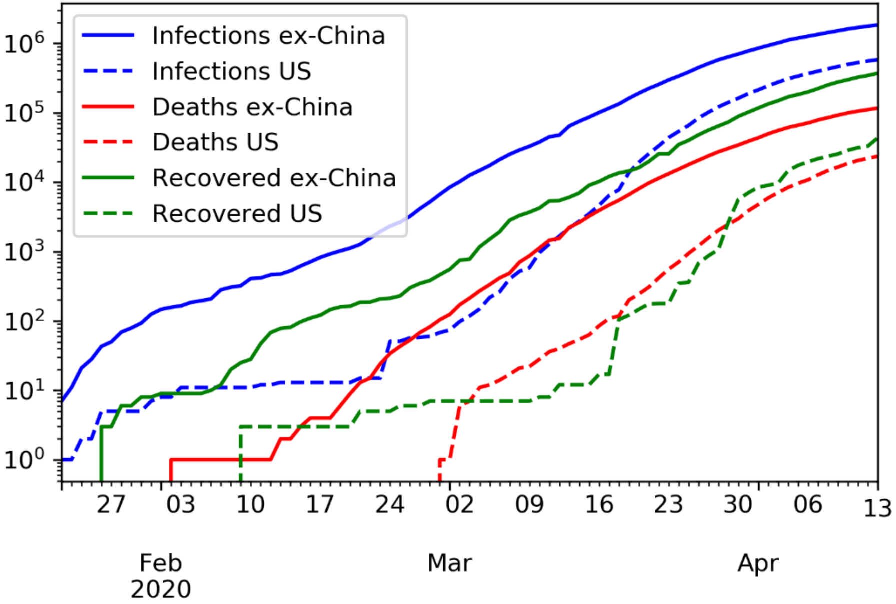
The Need for Exit Strategies

- **Government interventions have a disproportionate adverse effect on minorities, the working class, and the poor**
  - Layoffs have concentrated on low-income segments of the population
- Lockdowns will have significant secondary effects for years to come, in terms of
  - Drug abuse, poor education, family disintegration, etc.
- Lockdowns cannot be sustained until a vaccine is available (approx. 18 months)
- Accordingly, national governments must devise, communicate, and *justify* their exit strategies
  - **This study presents a general framework for analyzing the effects of alternative exit strategies**

The Data

Time Series per Country / Region

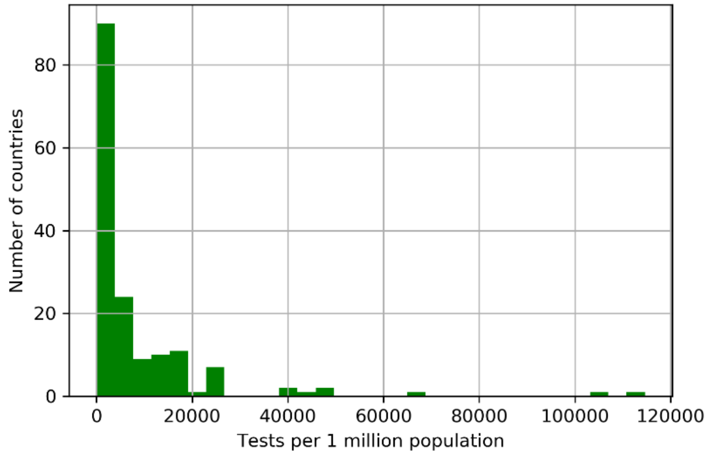
- The Johns Hopkins University Center for Systems Science and Engineering (CSSE) maintains a data repository, with daily counts per country/region of the following variables:
  - confirmed infections (C)
  - deaths (D)
  - recovered (R)
- The number of active cases (A) is implied as $A = C - (D + R)$
- Additionally, various national agencies report the time series of tests administered (T)



Time series of confirmed infections, deaths, and recovered, in logarithmic scale. In the U.S., the number of recovered cases is barely above the number of deaths, likely due to limited testing.

Problems in the Data

- COVID-19 is a health crisis aggravated by a data crisis
  - The number of *confirmed* infections is only a fraction of the number of infections
  - Tested individuals are primarily those where an infections is suspected
  - The number of tested individuals is lower than the number of tests administered
    - Individuals may require multiple tests in order to confirm a diagnosis
  - There is evidence that some countries have manipulated statistics
    - Among others, China, Indonesia, and Iran
- Flu pandemics are reoccurring events
  - 1918, 1957, 1968, 2009, 2020, …
- **COVID-19 is also a failure of planification**



After more than 5 weeks since the WHO declared COVID-19 a pandemic, we still do not have accurate statistics. Only a few countries have conducted well-designed statistical experiments to estimate the true values of C, D, R.

COVID-19 Fatality Rate

Case Fatality Rate

- Given the number of deaths D and the number of recovered individuals R, the maximum likelihood estimate of the *case fatality rate* d is

$$\hat{d} = \frac{D}{D + R}$$

  - That is, d is the ratio between deaths and resolved outcomes $(D + R)$

- This is different from the crude fatality ratio,

$$\hat{d}_c = \frac{D}{C}$$

  - The estimate of d_c is less useful, because some of the confirmed cases may not resolve favorably

(Chung Sung-Jun/Getty Images)

HEALTH

WHO Says The Coronavirus Global Death Rate Is 3.4%, Higher Than Earlier Figures

BI ROSIE PERPER, BUSINESS INSIDER
5 MARCH 2020

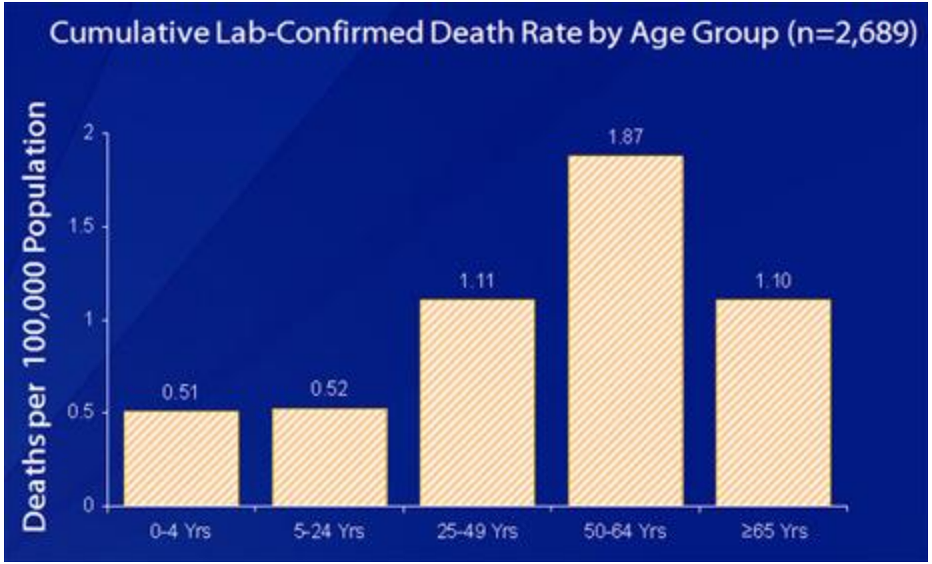
The latest global death rate for the novel coronavirus is 3.4 percent – higher than earlier figures of about 2 percent.

The coronavirus outbreak that originated in Wuhan, China, has killed more than 3,100 people and infected nearly 93,000 as of Tuesday. The virus causes a disease known as COVID-19.

On March 3, 2020, the Director-General of the WHO reported that COVID-19 had a crude fatality rate of 3.4%.

Previous Pandemic Example: The Swine Flu

- The Swine Flu pandemic of 2009 was caused by the H1N1 virus
- Ten weeks into the epidemic, estimates varied widely between countries, with case fatality rates reported between 0.1% and 5.1%
- It took years for doctors to realize that H1N1's case fatality rate was only 0.02%
- It is highly likely that WHO's original estimate of 3.14% is also wrong
  - The problem is that early statistics tends to underestimate the number of recovered individuals (R)



Source: CDC

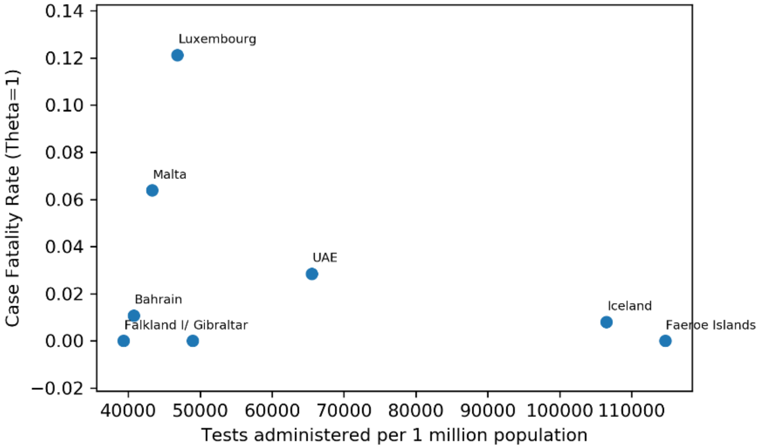
Flawed data led to a massive overestimation of the fatality rate of H1N1. Eleven years later, data collection is still a challenge.

Adjusted Fatality Rate

- The value of D can be measured with some confidence, because deceased individuals are tracked more carefully for legal reasons
- In contrast, only a fraction θ of recovered cases are confirmed, with $\theta \in [0,1]$, due to the overrepresentation of symptomatic cases in C
  - Asymptomatic patients have been systematically excluded from R
- Accordingly, the adjusted fatality rate is

$$\hat{d}_\theta = \frac{D}{D + \theta^{-1} R} \leq \hat{d}$$

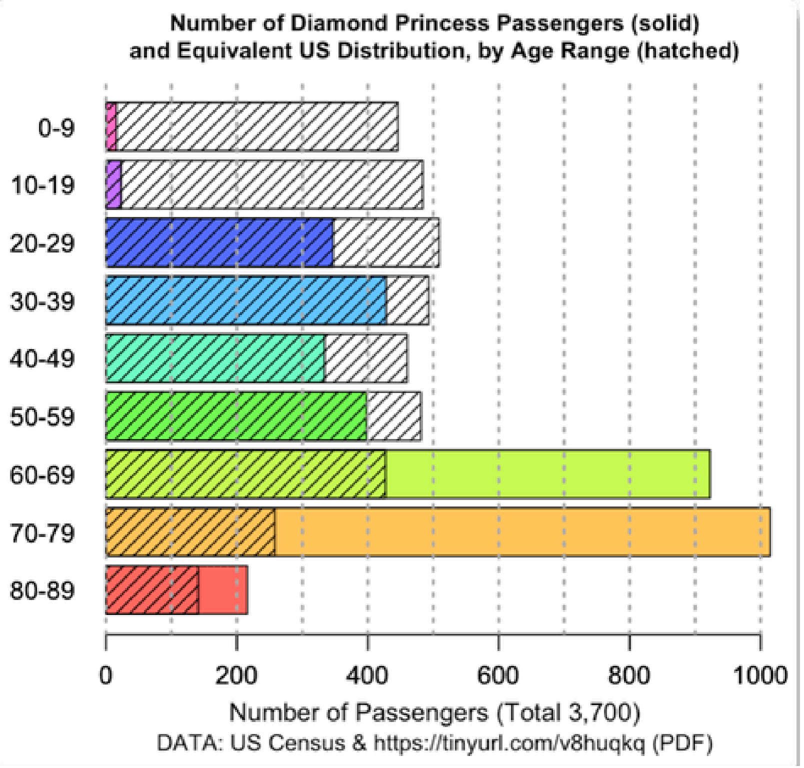
  - The Chinese CDC has estimated that the proportion of mild cases of COVID-19 is approximately 80.9%, thus globally $\theta < 0.191$



Countries that have administered tests to a broader portion of the population tend to report lower case fatality rates. This is consistent with the fact that $\theta \ll 1$.

Case Study: The Diamond Princess

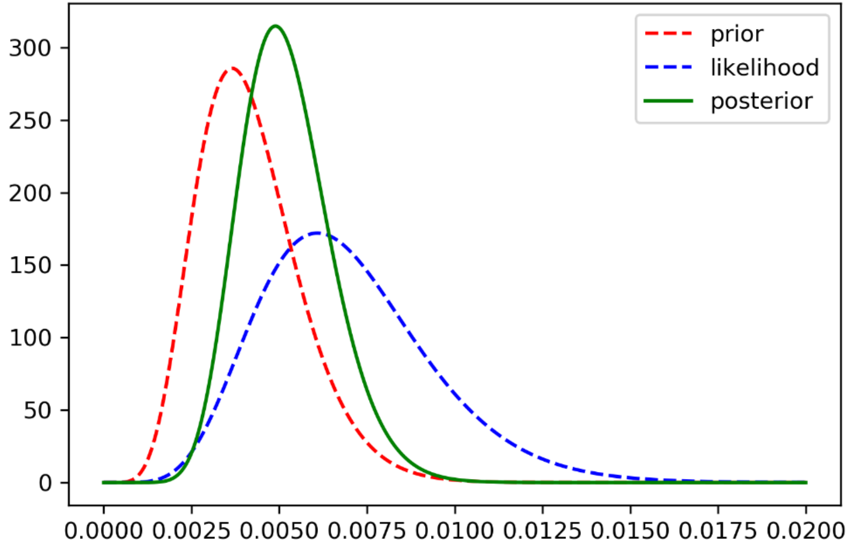
- Every passenger of the Diamond Princess has been tested ($\theta = 1$)
- In this case, $C = 712$, $D = 12$, and $R = 644$, implying a $\hat{d}_1 \approx 1.83\%$
- The problem with this estimate is that infected people were primarily elderly
  - The majority of passengers were between 70-79 years old
- We should expect that the global fatality rate for COVID-19 is much lower than 1.83%



Source: wattsupwiththat.com

Case Study: Iceland

- Iceland and the Faeroe Islands are the countries with most widespread testing relative to their population
  - So far, the Faeroe Islands has not registered a single death related to COVID-19 ($C = 185$)
  - Iceland has administered 38,204 tests on a population of approx. 341,243
  - Iceland has recorded values of $C = 1{,}739$, $D = 8$, and $R = 1{,}144$
- But even in the case of Iceland we must accept that $\theta < 1$, and $\theta \in [0.191,1]$
  - Less than 10% of the population has been tested
  - As a compromise, we use the middle of the range, $\theta = 0.6$

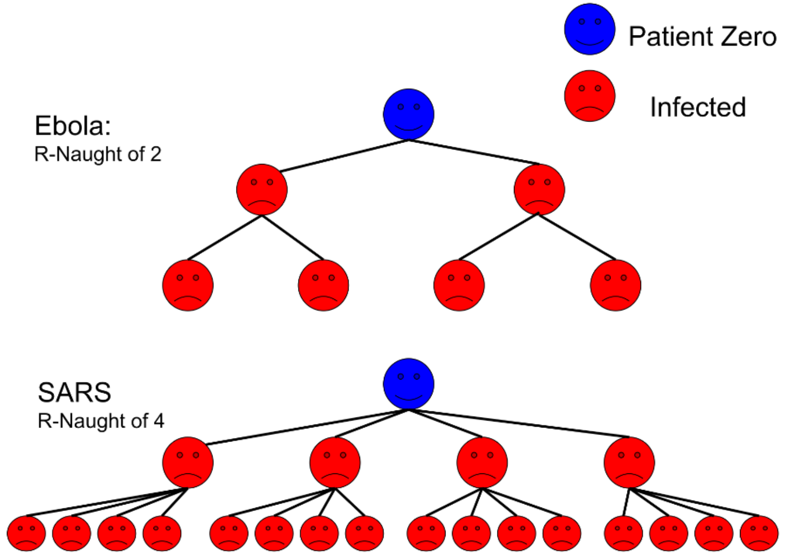


Using a likelihood function $Bi[D + R, \hat{d}_1]$, and a prior $Beta[D, \theta^{-1}R]$, **we derive a Bayesian estimate of $\widehat{d}_\theta \approx 0.42\%$ for COVID-19**, with 95% confidence bands of $[0.33\%, 0.75\%]$. This is significantly lower than WHO's estimate of 3.14%.

COVID-19 Reproductive Numbers

Basic vs. Effective Reproductive Numbers

- The effective reproductive number ($\Re_1$) is the number of infections caused by a single individual when some members of the community have developed immunity, or some intervention measures are in place
- The basic reproductive number ($\Re_0$) is the effective reproductive number when there is no immunity from past exposures or vaccination, nor any deliberate intervention in disease transmission



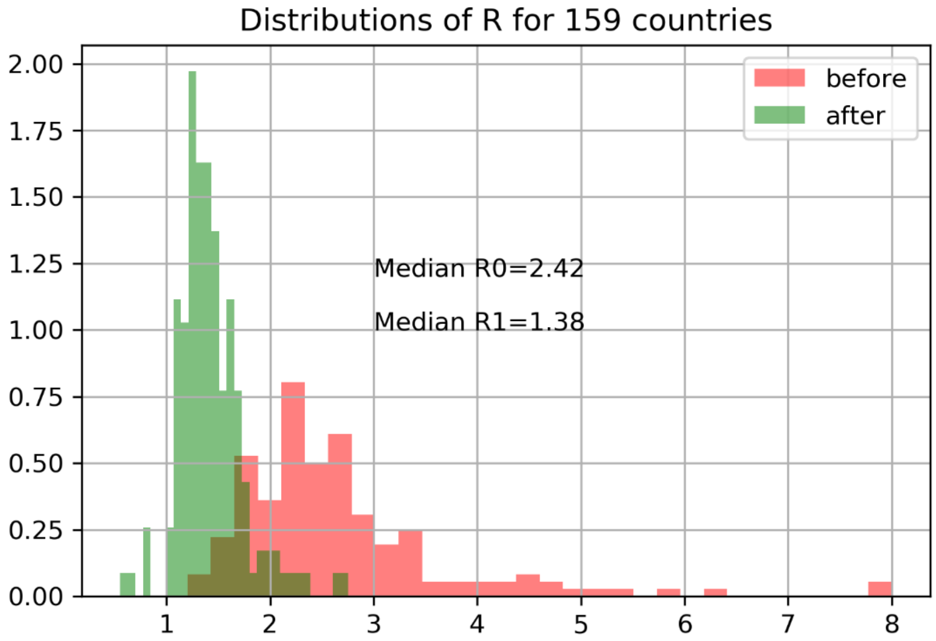
Source: Wikipedia

Illustration of basic reproductive numbers for Ebola ($\Re_0$=2) and SARS ($\Re_0$). Past immunity and interventions help achieve $\Re_1 < \Re_0$.

Estimating Reproductive Numbers for COVID-19

- We can derive the $\Re_0$ and $\Re_1$ values by [fitting the SIR model]()
  - Doctors at the frontlines tell us that the median duration of infectiousness is approx. 7 days. We do not need to estimate γ^{-1}
- Given a time series of observations $\{s_t, i_t, r_t\}_{t=1,\ldots,T}$, we can determine $\hat{\beta}$ by solving the equation $\Delta i_{t+1} + \gamma i_t = \beta s_t i_t + \varepsilon_t$, with solution

$$\Re = \frac{\sum_{t=1}^{T} (\Delta i_{t+1} + \gamma i_t) s_t i_t}{\gamma \sum_{t=1}^{T} (s_t i_t)^2}$$

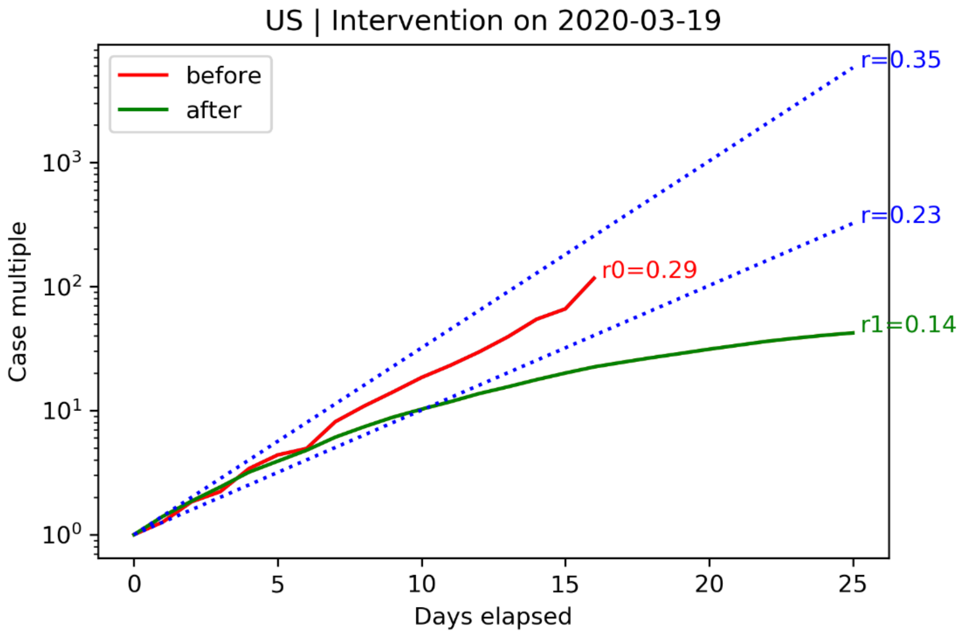


The distribution of $\Re_0$ and $\Re_1$ across 159 countries shows a steep decline in reproductive numbers following government interventions.

Lockdown Effectiveness

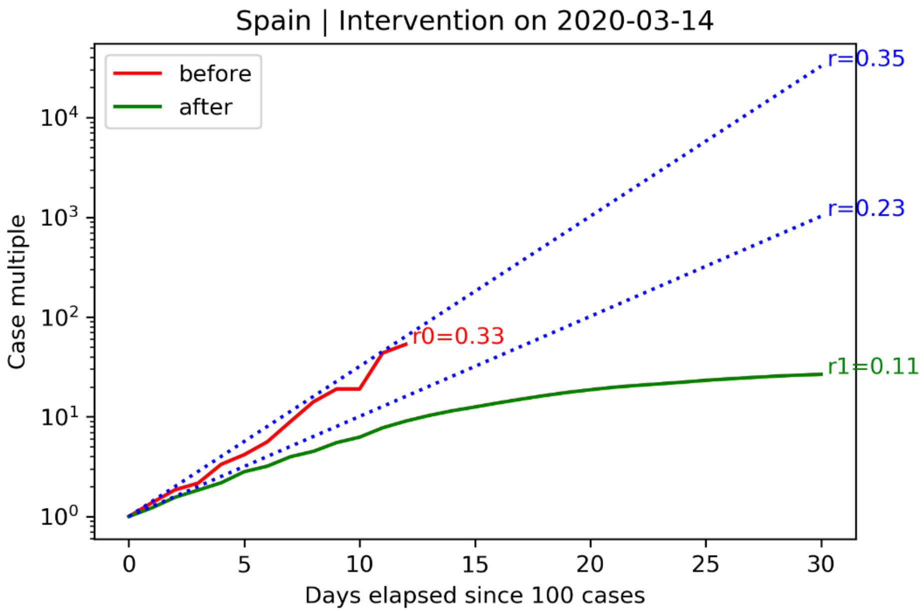
Measuring Growth Rates Before and After

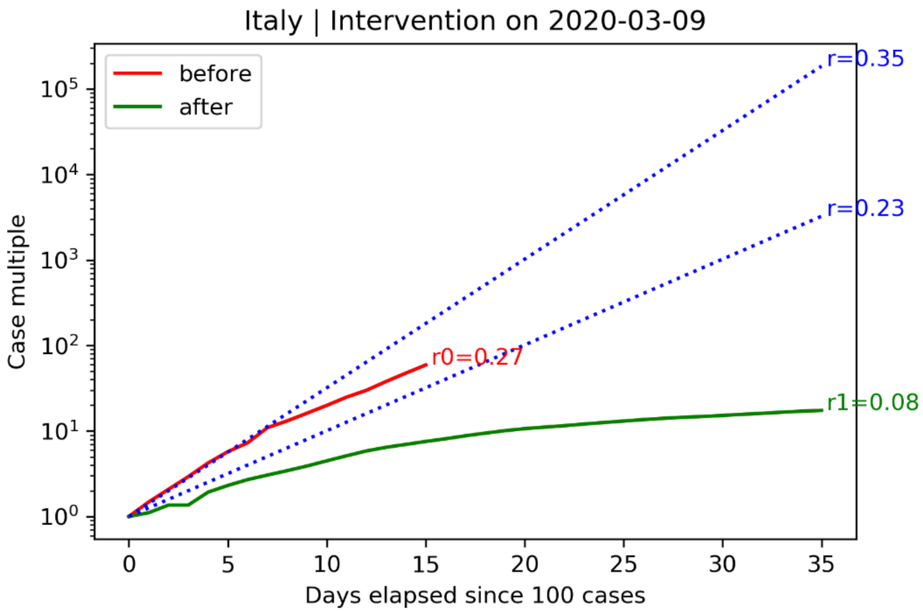
- We can evaluate the effectiveness of lockdowns in terms of confirmed infections before and after measures were adopted
- The right plot shows the rate of exponential growth in C before and after government lockdowns
  - Data before $C \geq 100$ is dropped (size is too small for an epidemic)
- The blue lines indicate expected growth when
  - cases double every 2 days ($r = 0.35$)
  - cases double every 3 days ($r = 0.23$)



Before the government intervened, cases grew in the U.S. exponentially, at a growth rate of 0.29. Even after lockdowns, it took 10 days for cases to fall below the double-every-3-days line. Benefits are not instantaneous.

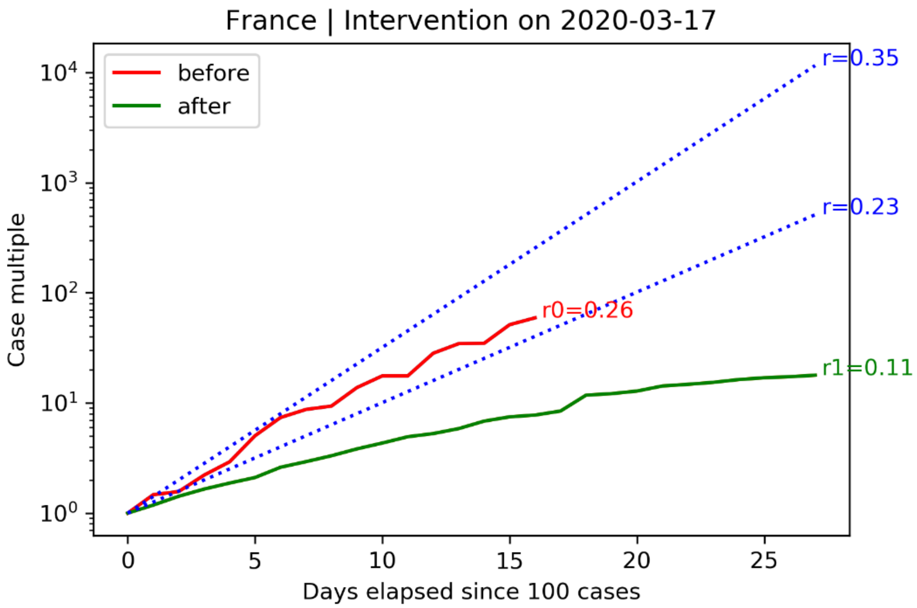
Countries with the Largest Number of Cases (1/3)

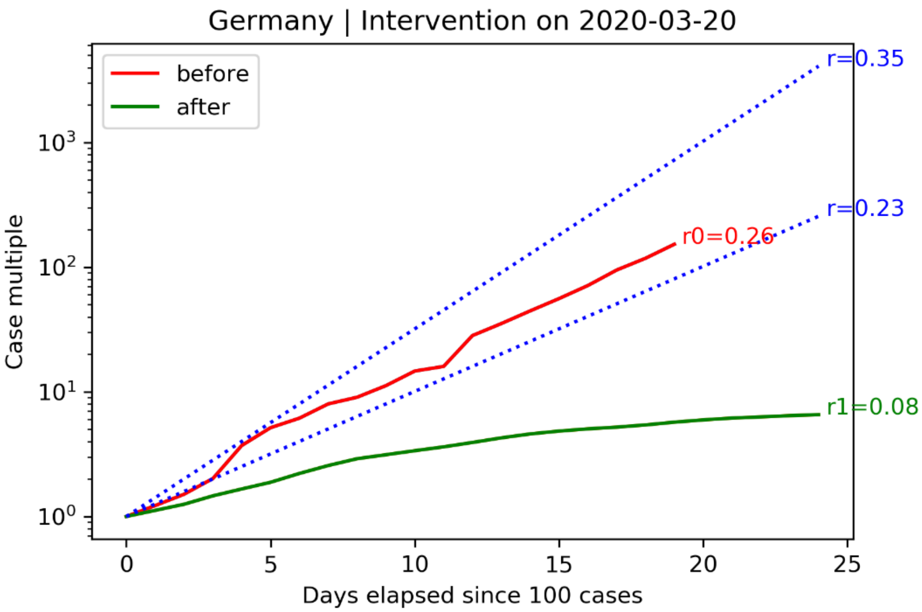




Spain and Italy have economies that rely heavily on tourism and public transport. The situation in Spain was particularly alarming, with cases almost doubling every 2 days. Government mandated lockdowns successfully curved the spread of the disease.

Countries with the Largest Number of Cases (2/3)

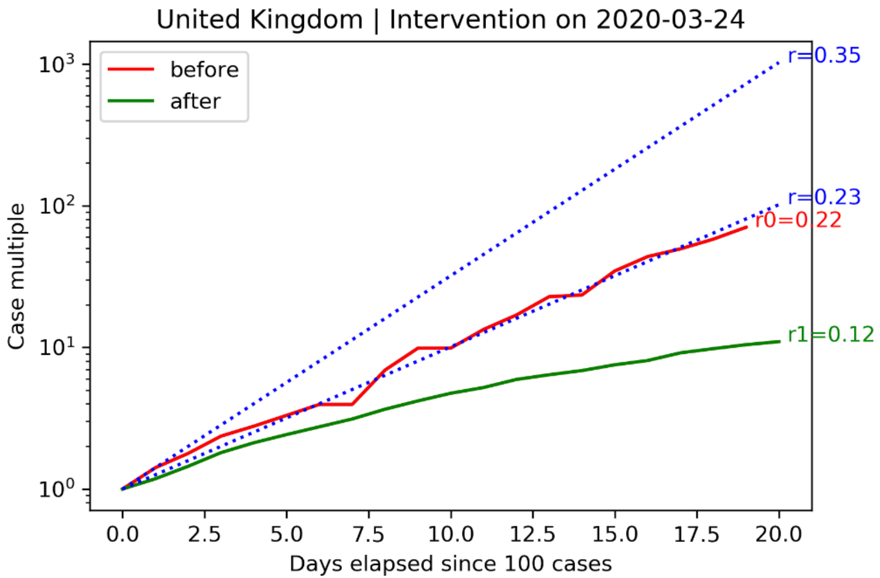




France and Germany also experienced exponential growth. Their governments were able to tame the spread of the disease without resorting to the drastic lockdowns enacted in Spain and Italy.

The United Kingdom

- The United Kingdom did not declare a National Emergency until March 24, 2020
- The initial situation was slightly better than in other European countries
  - Cases doubled every 3 days before government lockdowns
  - After 18 days, the spread of the disease did not slow down, forcing the UK government to ban gatherings of more than 2 people
- After measures were adopted, the spread of the disease slowed down

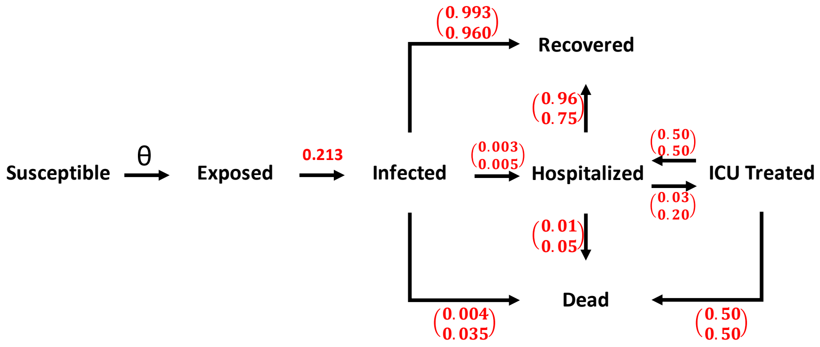


UK's COVID-19 exponential growth rate of $r = 0.12$ is in line with other European countries.

COVID-19 Exit Strategies

Parameters for Scenario Simulations

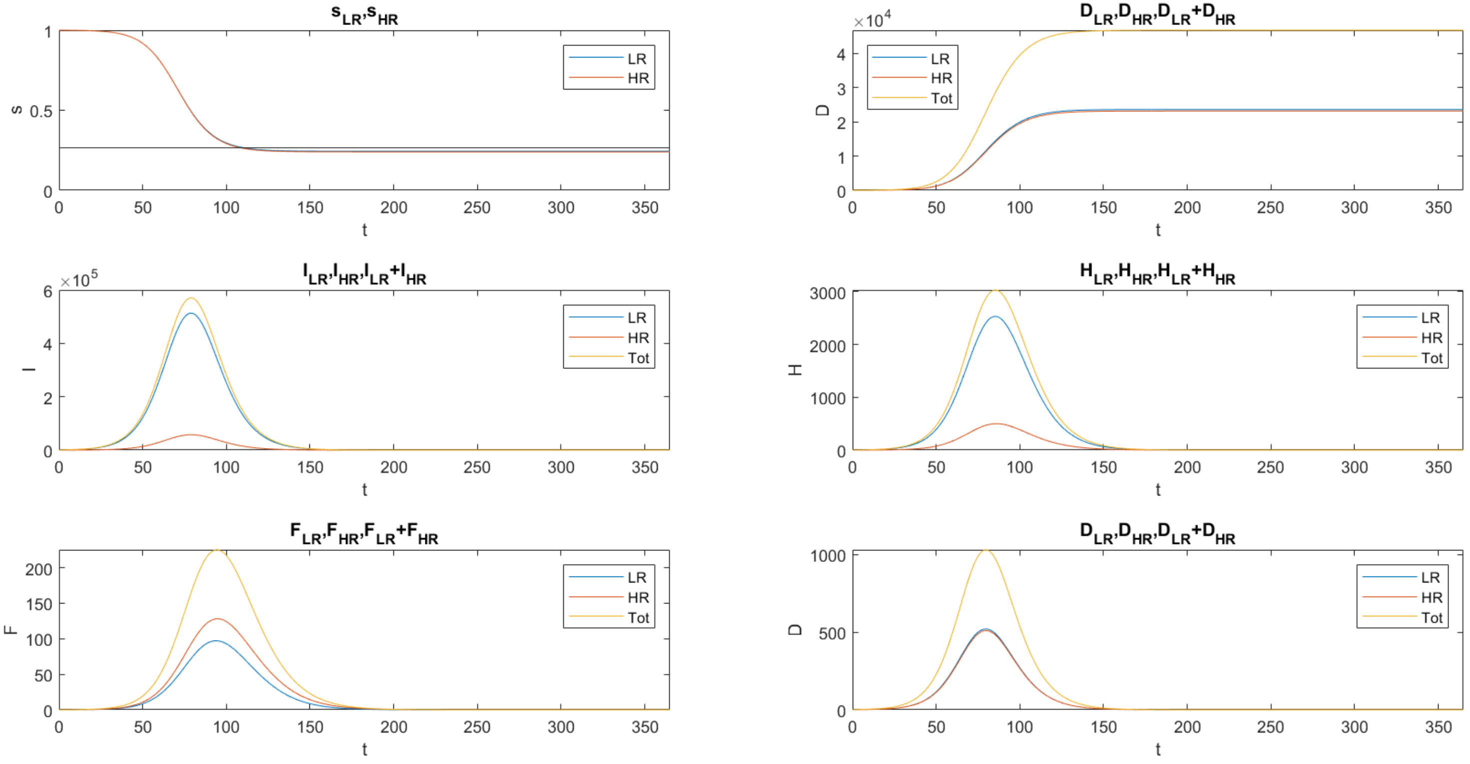
- Total population is 8,500,000 (e.g., NYC)
  - 90% is at low risk (LR), 10% at high risk (HR)
- Reproductive numbers
  - $R_1 = 1.8 \pm 0.2, R_2 = 1.8 \pm 0.2$ without lockdown
  - $R_1 = (1.8 \pm 0.2) * 0.6, R_2 = (1.8 \pm 0.2) * 0.55$ during the lockdown
- Relative size of interactions btw groups
  - $\pi_1 = 0.9, \pi_2 = 0.7$
- Other parameters are shown on the right



Parameters used in all scenarios. The details of the K-SEIR Model are explained in the Appendix.

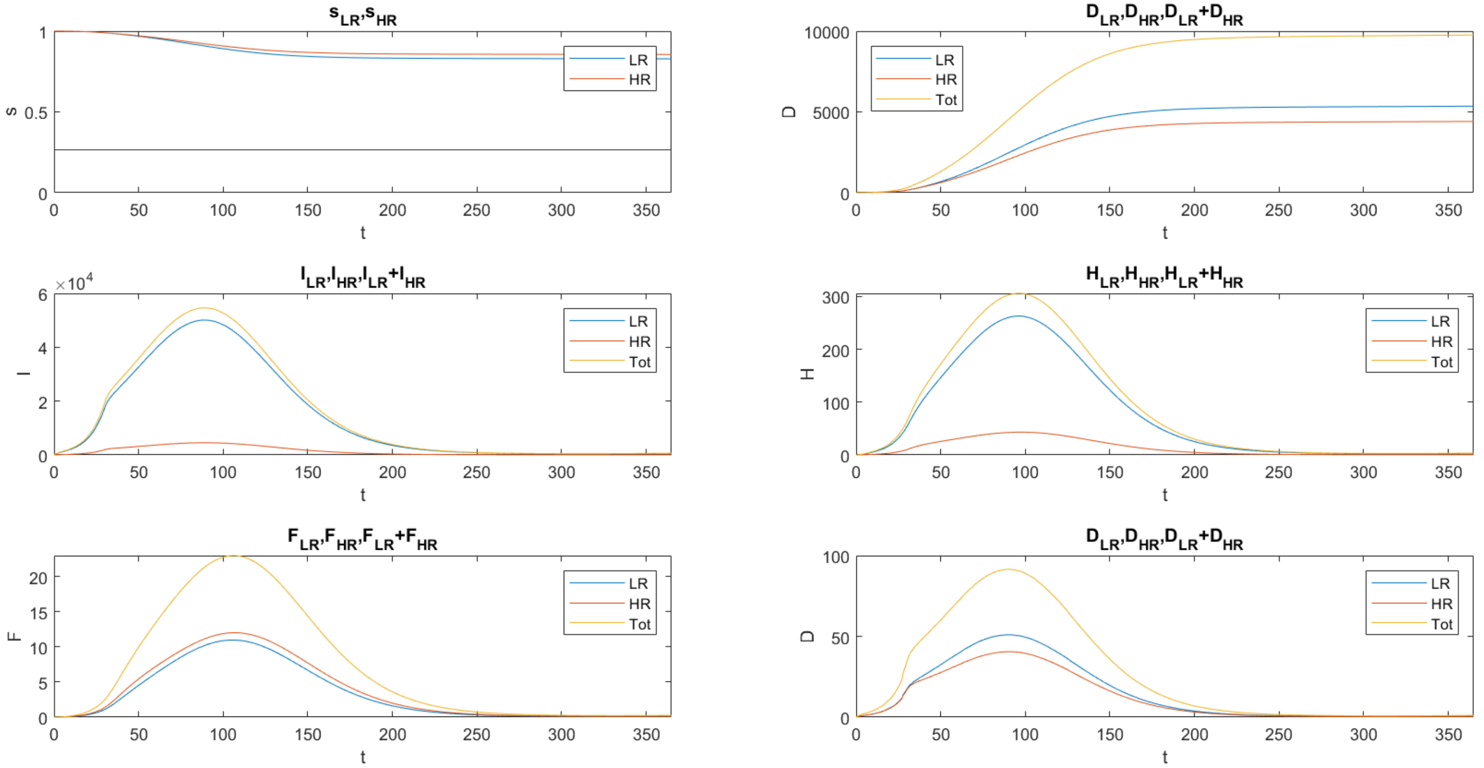
The following figures demonstrate daily dynamics for fraction of susceptibles for low risk and high risk (s_{LR}, s_{HR}); number of infected ($I_{LR}, I_{HR}, I_{LR} + I_{HR}$); hospitalized ($H_{LR}, H_{HR}, H_{LR} + H_{HR}$); on ventilators ($F_{LR}, F_{HR}, F_{LR} + F_{HR}$); deaths ($D_{LR}, D_{HR}, D_{LR} + D_{HR}$); and daily cumulative deaths in upper right corner

Strategy 1: No Intervention



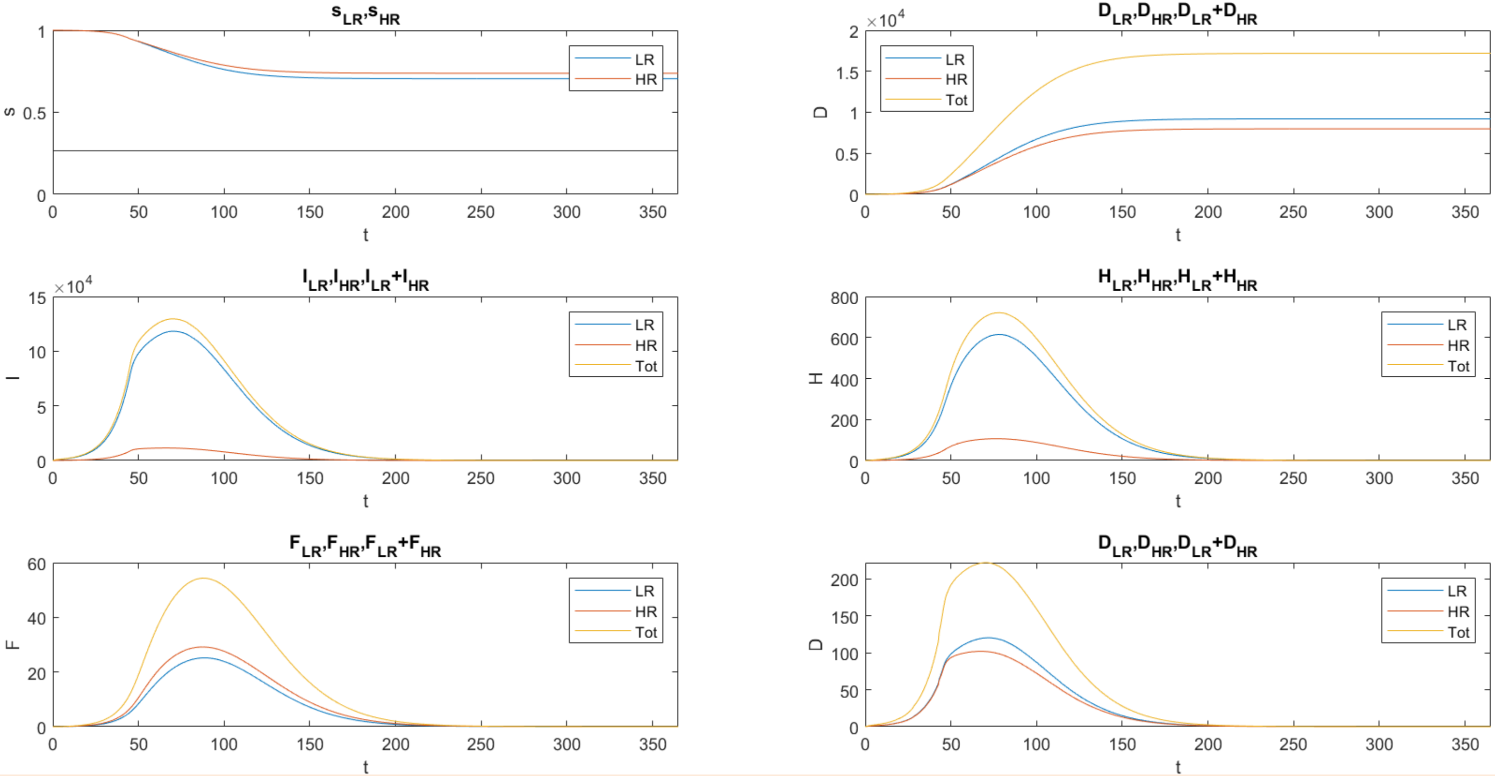
Disease dynamics in the case of no intervention with enough hospital beds and ICU units. Around 75% of LR and 75% of HR groups are infected after 365 days. Total number of deaths is 46,710.

Strategy 2a: Lockdown starts after 30 days



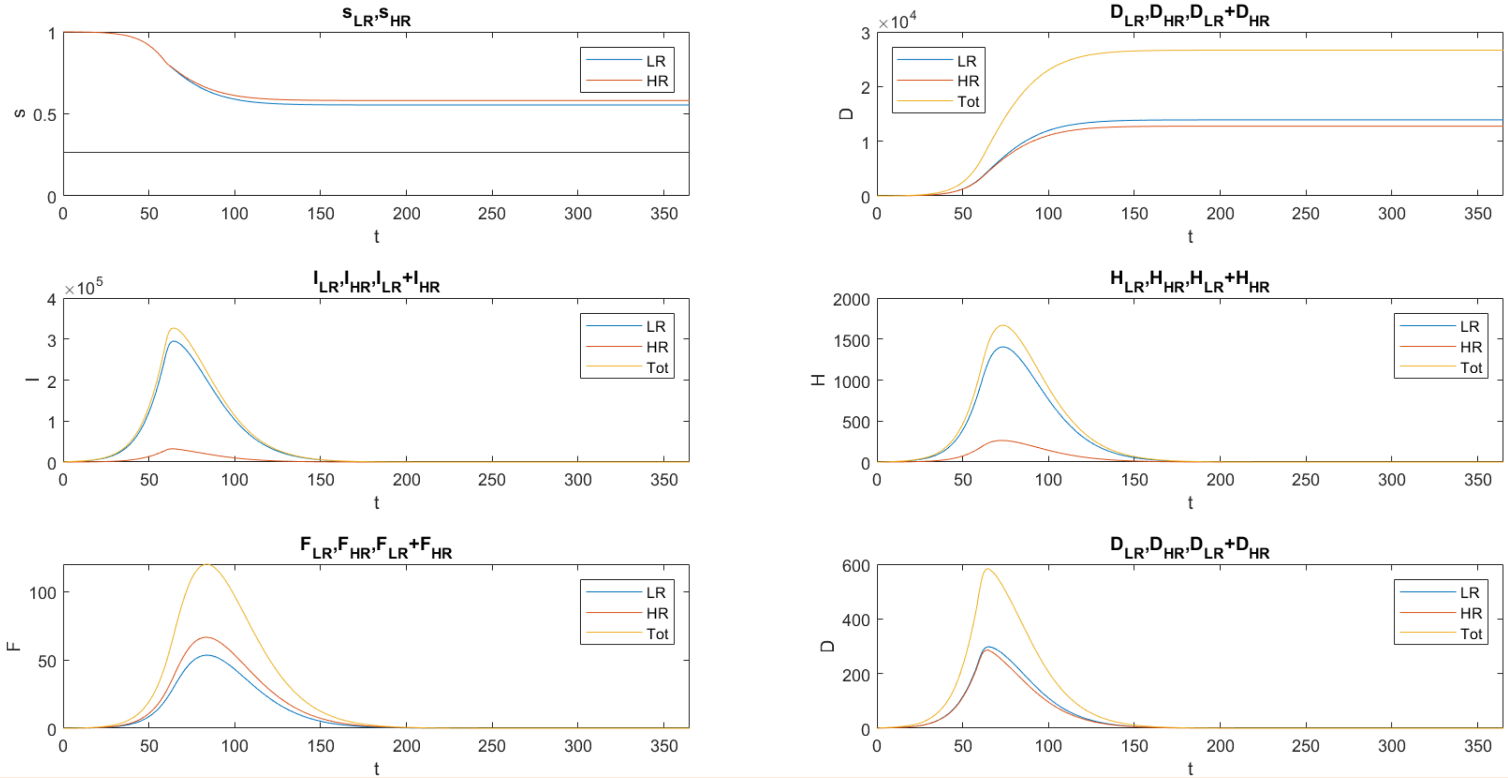
Disease dynamics in the case of lockdown 30 days after first case. Around 10% of LR and 10% of HR groups are infected after 365 days. The total number of deaths is 9,710.

Strategy 2b: Lockdown starts after 45 days



Disease dynamics in the case of lockdown 45 days after first case. Around 20% of LR and 20% of HR groups are infected after 365 days. The total number of deaths is 17,190.

Strategy 2c: Lockdown starts after 60 days

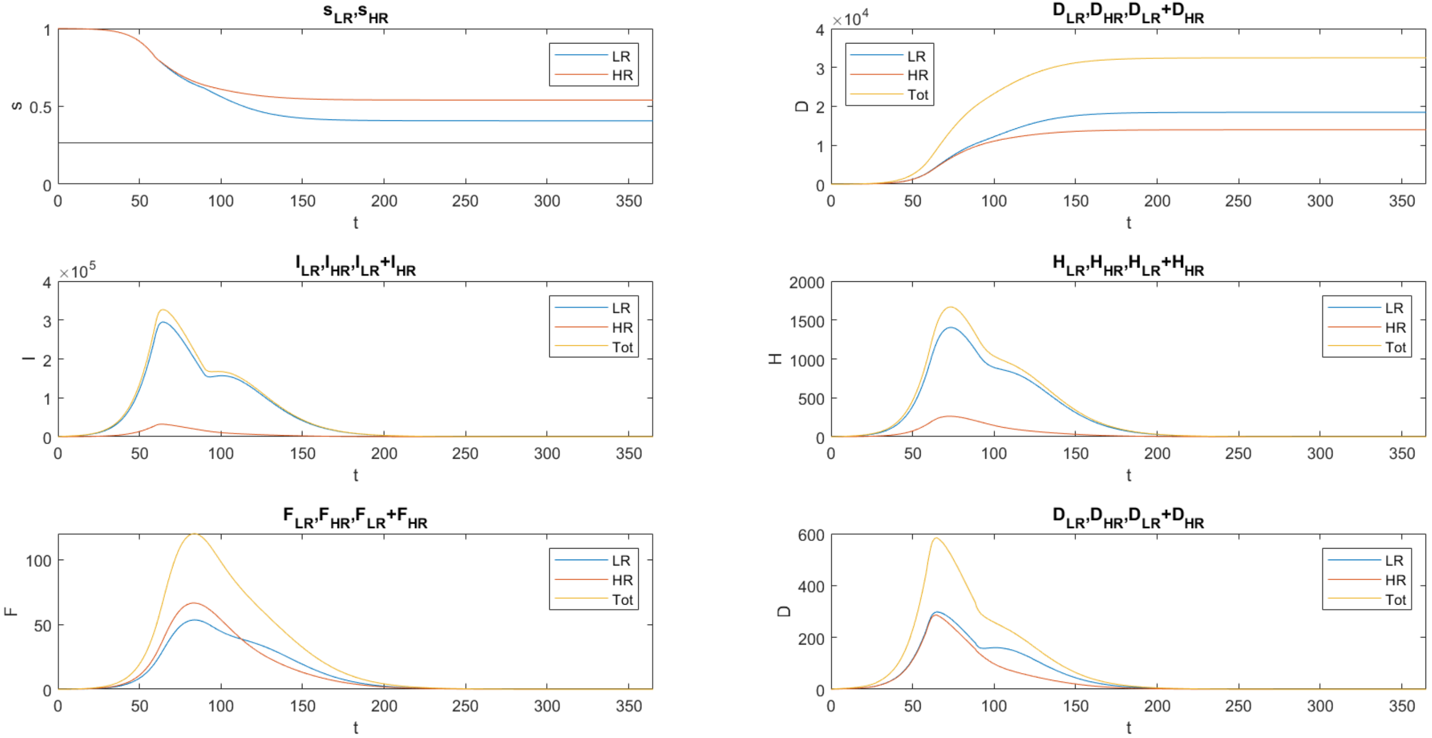


Disease dynamics in the case of lockdown 60 days after first case. Around 50% of LR and 50% of HR groups are infected after 365 days. The total number of deaths is 26,660.

Strategies for Lifting Lockdowns

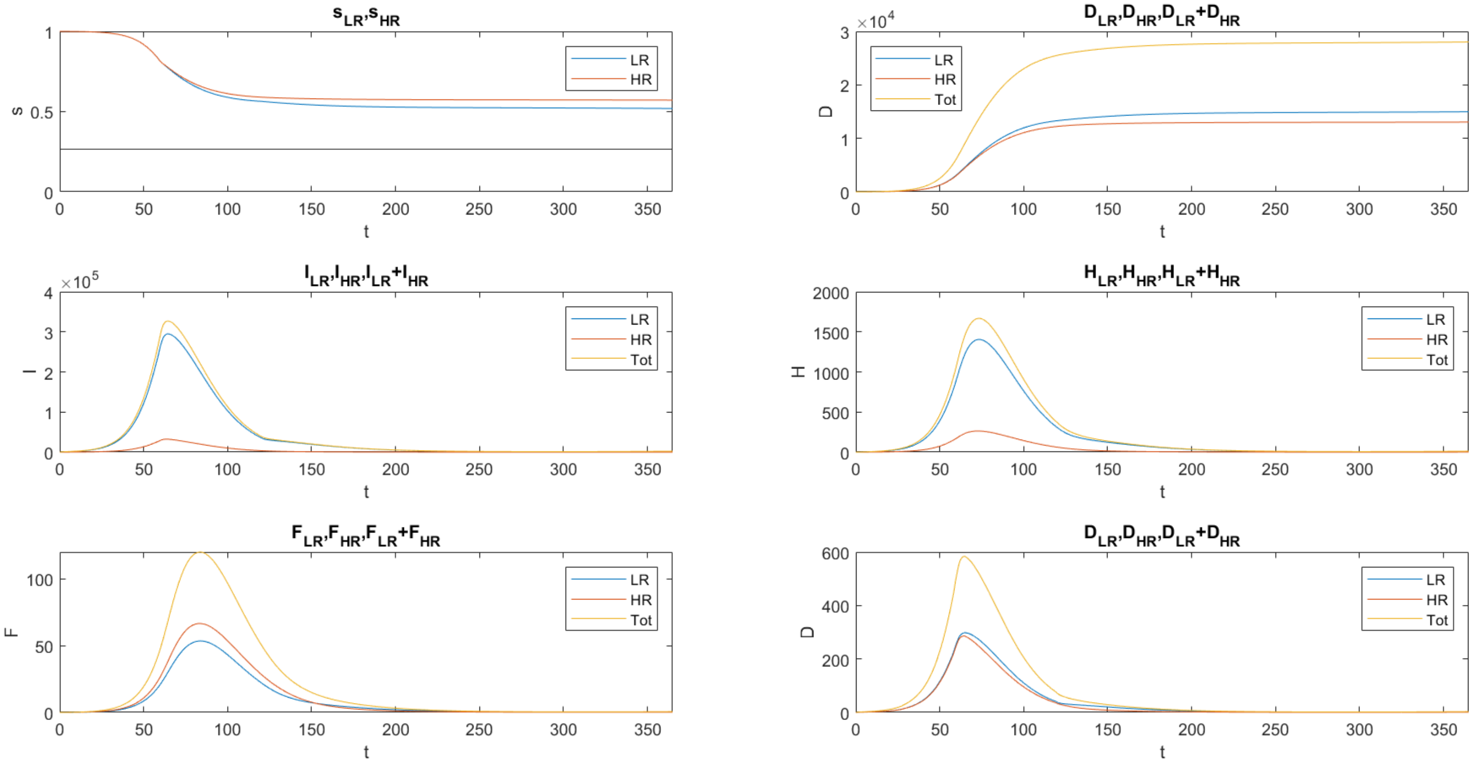
- **Early detection of disease and intervention is the most efficient way of dealing with pandemics in such densely populated areas as New York city**
  - The experience with Los Angeles is another good example
- However, because of the difficulties with data collection and other considerations, this is often impossible
- We consider several strategies for lifting the lockdown, assuming the lockdown starts 60 days after the first case was detected:
  - Lockdown lasts 30 or 60 days, and
  - Lockdown is lifted on low-risk group only, or
  - Lockdown is lifted on both groups
- These strategies are compared to the base case **Strategy 2c**

Strategy 3a: Lockdown lifted after 30 days on LR



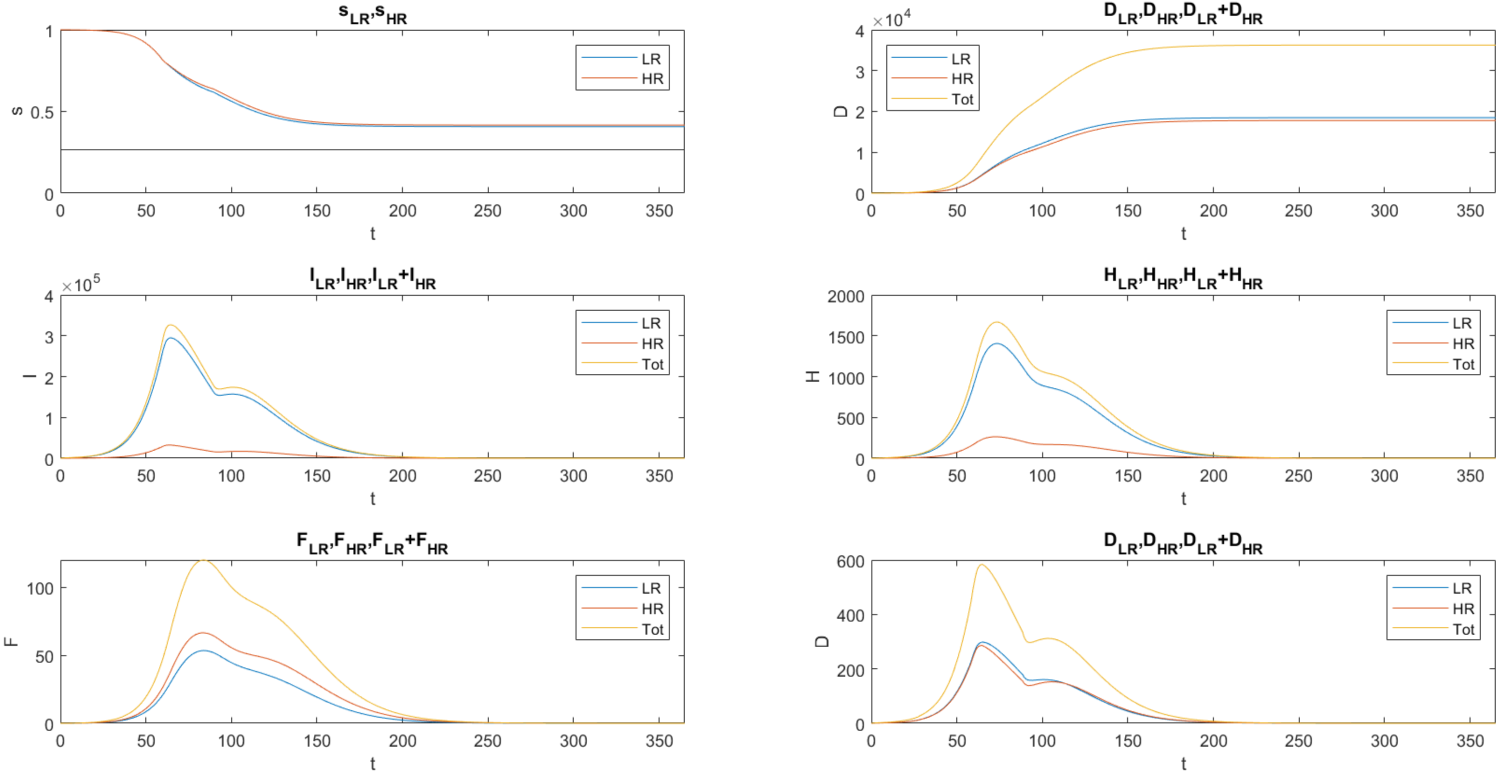
Disease dynamics when lockdown on low-risk group is lifted after 30 days. Proportion of infected HR individuals after 365 days remains at about 50%, while proportion of infected LR increases to 60%. Total deaths increase from 26,660 to approximately 32,470.

Strategy 3b: Lockdown lifted after 60 days on LR



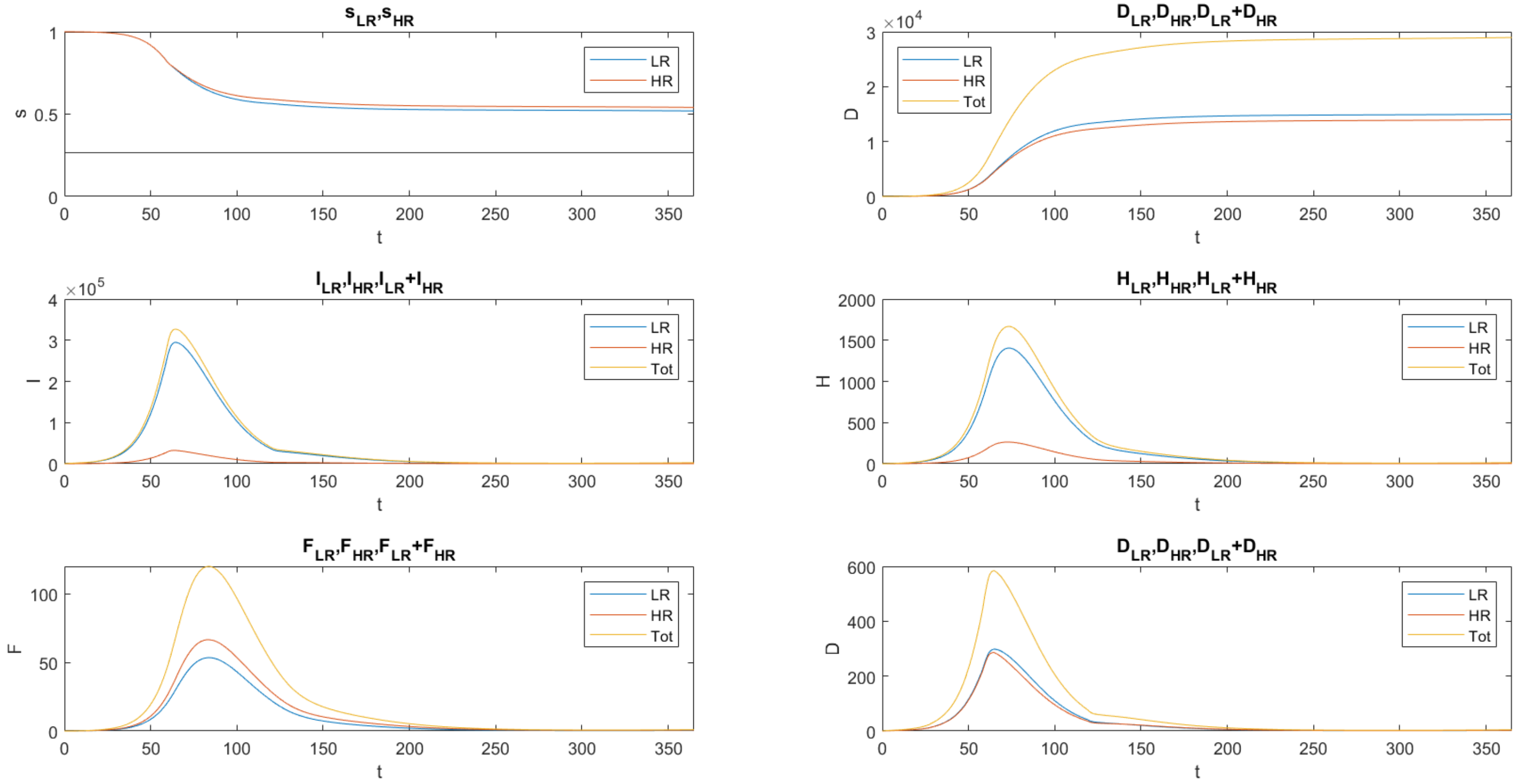
Disease dynamics when lockdown on low-risk group is lifted after 60 days. Proportion of infected individuals after 365 days remains at about 50% in both group. Total deaths increase from 26,660 to approximately 28,000.

Strategy 4a: Lockdown lifted after 30 days on All



Disease dynamics when lockdown on both groups is lifted after 30 days. Proportion of infected individuals in both groups after 365 days increases to about 60%. Total deaths increase from 26,660 to approximately 36,270.

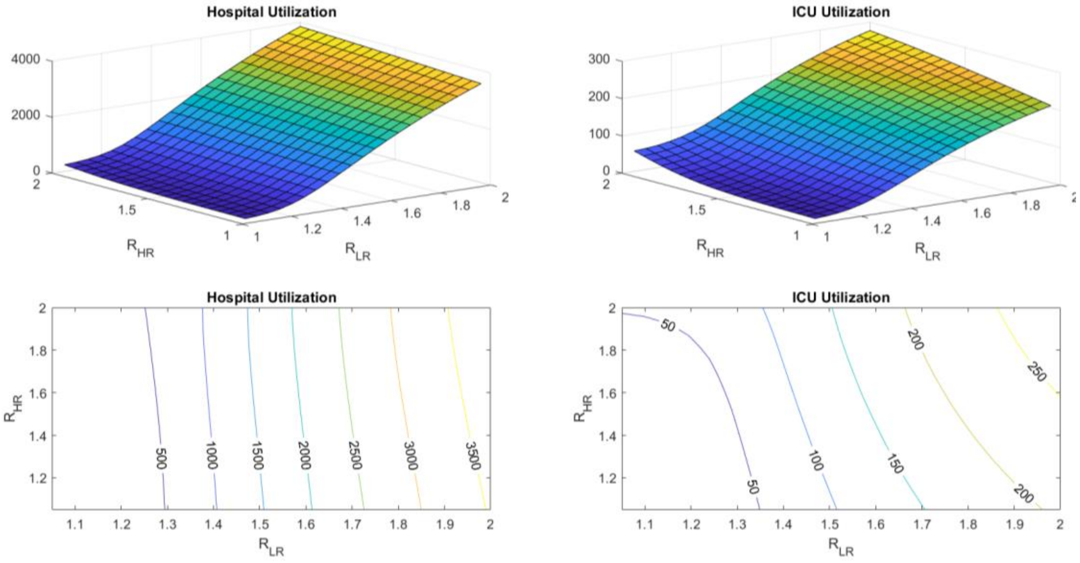
Strategy 4b: Lockdown lifted after 60 days on All



Disease dynamics when lockdown on both groups is lifted after 60 days. Proportion of infected individuals after 365 days remains at about 50% in both group. Total deaths increase from 26,660 to approximately 28,950.

Hospital and ICU Utilization (1/2)

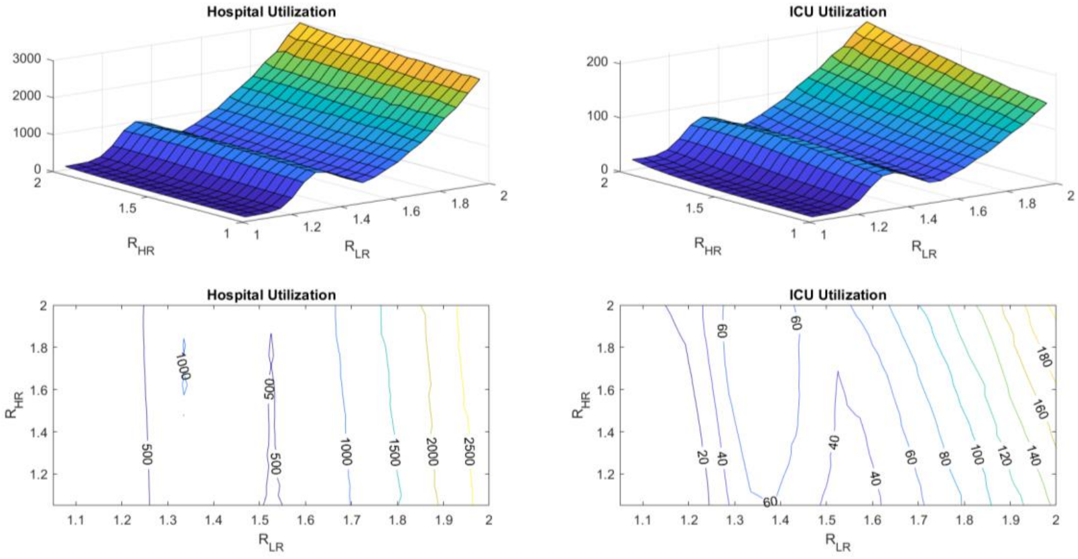
- In this graph we show maximum hospital and ICU utilization as functions of R_1 and R_2
- We assume that no lockdown is imposed



Hospitalization and ICU usage without lockdown as a function of the original reproductive numbers.

Hospital and ICU Utilization (2/2)

- In this graph we show maximum hospital and ICU utilization as functions of R_1 and R_2
- We assume that a 60-day lockdown is imposed 60 days after the first case



Hospitalization and ICU usage with 60-days lockdown as a function of the original reproductive numbers.

Conclusions

Main Conclusions (1/2)

- Governments need protocols for “nowcasting” the mortality and transmissibility of the pandemic
  - For example, a random sample of 1,000 individuals would have quickly dispelled WHO’s assertion that COVID-19 had a fatality rate of 3.4%
- Government statistics show that lockdowns have been successful at
  - slowing down the spread of COVID-19
  - reducing the number of deaths directly caused by this disease
- However, government statistics do not account for the loss of lives and livelihoods derived from universal lockdowns
  - **The objective of lockdowns should be to minimize the total loss of life, not only deaths directly caused by the pandemic**
  - In particular, large-scale unemployment is a leading cause of drug abuse. **Over the next months and years, we will likely observe a spike in drug-abuse related deaths, crime, and mental health issues**

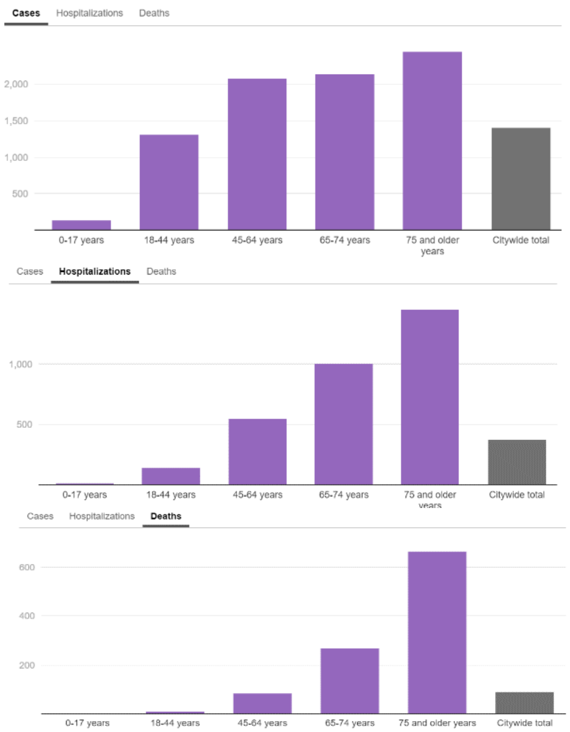
Main Conclusions (2/2)

- **The K-SEIR model shows that targeted lockdowns (on high-risk populations) are more likely to achieve the triple goal of**
  - minimizing loss of lives
  - minimizing loss of livelihood
  - avoiding a depletion of medical resources
- **A brief universal lockdown is warranted while we collect data regarding mortality and transmissibility, followed by targeted lockdowns of the high-risk population**
  - Low-risk population should continue to practice sensible personal protection measures
- **Governments must learn from the mistakes of COVID-19's crisis management, and design targeted lockdowns in anticipation of COVID-20**
  - There is not one size that fits all, and national governments must device tailored targeted lockdowns based on their particular circumstances

Appendix: The K-SEIR Model

The Standard SEIR Model

- The famous Susceptible-Exposed-Infected-Recovered (SEIR) model, originated by Kemrack and McKenrick in 1927, is the main workhorse of the mathematical epidemiology
- **The model is adequate when the entire pool of the susceptible population reacts similarly to the infection**
- However, COVID-19 hits different population groups differently
  - We must extend the SEIR model, so that it models the dynamics of different pools of individuals
- **We propose the heterogenous version of the SEIR model, with K groups: The K-SEIR model**

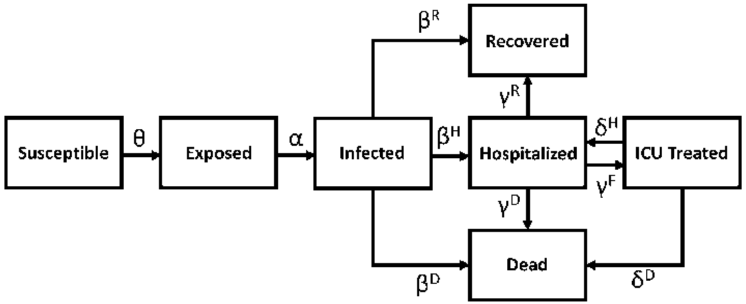


Source: nyc.gov

Cases, Hospitalizations, and Deaths by age.

The K-SEIR Model

- To account for different impact of the disease on different groups, we need to split the entire population into K groups
  - In the context of COVID-19, age is clearly an important determinant of which group an individual belongs to, but other factors must be considered, such as pre-existing conditions and obesity
- To properly describe various stages of the disease, we need to consider several state transitions within a given group
- Specifically, we need to study susceptible, exposed, infected, hospitalized, ICU treated, recovered, and dead subgroups within a class



The flow chart of a SEIR model within a given class.

Main K-SEIR Equations

- Let S_k, E_k, I_k, H_k, F_k, R_k, D_k, be the number of susceptible, exposed, infected, hospitalized, ICU treated, recovered, and dead individuals in the k-th group, and N_k be the total number of people at a given moment in time
- Susceptible members become exposed upon contact with infected members, exposed become infected after a period, they stay infected for a while, and then either recover, or die, or admitted to the hospital
- Similar stages take place after hospitalization, as per the previous diagram
- In a sense, we are dealing with a version of Kirchhoff's law

$$
\begin{aligned}
\frac{dS_k}{dt} &= -\Theta_k a_k S_k, \\
\frac{dE_k}{dt} &= \Theta_k a_k S_k - \alpha_k E_k, \\
\frac{dI_k}{dt} &= \alpha_k E_k - \left(\beta_k^{(H)} + \beta_k^{(R)} + \beta_k^{(D)}\right) I_k, \\
\frac{dH_k}{dt} &= \beta_k^{(H)} I_k - \left(\gamma_k^{(F)} + \gamma_k^{(R)} + \gamma_k^{(D)}\right) H_k + \delta_k^{(H)} F_k, \\
\frac{dF_k}{dt} &= \gamma^{(F)} H_k - \left(\delta_k^{(H)} + \delta_k^{(D)}\right) F_k, \\
\frac{dR_k}{dt} &= \beta_k^{(R)} I_k + \gamma_k^{(R)} H_k, \\
\frac{dD_k}{dt} &= \beta_k^{(D)} I_k + \gamma_k^{(D)} H_k + \delta_k^{(D)} F_k, \\
\frac{dN_k}{dt} &= -\left(\beta_k^{(D)} I_k + \gamma_k^{(D)} H_k + \delta_k^{(D)} F_k\right).
\end{aligned}
$$

The main equations for the multi-group case.

Explanation of Parameters and their Choice

- The transition coefficients α,β,γ,δ are inversely proportional to the average time spent by a representative individual in different states
  - These times are denoted by $\tau_E, \tau_I, \tau_H, \tau_F$
- For example, in the case of COVID-19, the most recent data suggests that
  - $\tau_E = 4.6\ days, \tau_I = 4\ days, \tau_H = 7 days, \tau_F = 10\ days$
  - We shall see that K-SEIR's estimates for COVID-19 are relatively robust to changes in these values
- The actual split of α,β,γ,δ between classes is somewhat nuanced. It can be inferred from the clinical information

$$\alpha, \quad \beta_k = \beta_k^{(H)} + \beta_k^{(R)} + \beta_k^{(D)},$$

$$\gamma_k = \gamma_k^{(H)} + \gamma_k^{(R)} + \gamma_k^{(D)}, \ \delta_k = \delta_k^{(H)} + \delta_k^{(D)}$$

$$\tau_E = 4.6 \text{ days}, \quad \tau_I = 4 \text{ days},$$

$$\tau_H = 7 \text{ days}, \quad \tau_{ICU} = 10 \text{ days}.$$

Representative time scales.

Contacts between Groups

- In order to describe contacts between groups, we need to introduce the so-called preferred mixing matrix C
- Our choice of matrix C allows us to consider all kind of possibilities, including the limiting cases when there is proportional mixing between groups (the matrix is degenerate, has rank one, and $\pi_k = 0$), or there is no mixing between groups at all (the matrix is diagonal , and $\pi_k = 1$)
  - An adequate choice of the mixing matrix is important for what we want to accomplish in our analysis of various lockdown strategies

$$C_{kl} = \begin{cases} \dfrac{(1-\pi_k)(1-\pi_l)a_k N_k a_l N_l}{\sum_{k=1}^{K}(1-\pi_k)a_k N_k}, & l \neq k, \\ \left(\pi_k a_k N_k + \dfrac{((1-\pi_k)a_k N_k)^2}{\sum_{k=1}^{K}(1-\pi_k)a_k N_k}\right), & l = k. \end{cases}$$

The general preferred mixing matrix.

Propagation of Infection

- In view of our choice of the preferred mixing matrix, we can describe the impact of infected individuals in the k-th group on susceptible individuals on all the groups in a proportional fashion
- In our framework, the celebrated **reproductive number**, which represents how many members of the public can be infected by a representative infected in a completely susceptible population, is given by the following ratio:

$$R_k = a_k/\beta_k$$

- This number is non-dimensional
  - For the common flu, R_k is of order 1.2-1.5
  - For some highly infectious diseases it can be as high as 10

$$\Theta_k = \pi_k \frac{I_k}{N_k} + (1-\pi_k)\frac{\sum_{k=1}^{K}(1-\pi_k)a_k I_k}{\sum_{k=1}^{K}(1-\pi_k)a_k N_k}.$$

The impact of infected on susceptible individuals.

Nonlinear Effects

- Our main equations are manifestly scale invariant
  - It means that all the outputs for a metropolitan area with 10M population are 10 times larger than the outputs for a metropolitan area with 1M population
- However, for large metropolitan areas it is more appropriate to use equations which violate scale invariance
  - This fact is well-known and comes as no surprise (public transportation, especially subway, being the main suspect)
- This nonlinearity explains why mortality in NYC is so much higher that in sparsely populated areas with similar population sizes
- To model nonlinear effects, we choose θ accordingly
  - For example, ϖ could be a factor of order 0.01, which accounts for the density of the metropolitan area

$$\Theta_k = \pi_k \frac{I_k}{N_k^{1-\varpi}} + (1-\pi_k) \frac{\sum_{k=1}^{K} (1-\pi_k) a_k I_k}{\sum_{k=1}^{K} (1-\pi_k) a_k N_k^{1-\varpi}},$$

The choice of θ

Description of the Lockdown

- The lockdown effects can be modelled by the reduction of the reproductive number, R_k
- Since in addition to these effects we also want to model seasonality, which is a common feature of many viral infections, we allow for R_k to be time dependent
- The lockdown starts at time T_1, and ends at time T_2
- Typically, the reduction of R_k as a result of a lockdown is in the order of 30%-40%

$$R_k(t) = \phi_k(t)\,\chi_k(t)\,R_k^{(0)},$$

$$\phi_k(t) = \bar{\phi}_{k,l}\mathcal{H}\left((t - t_{l-1})(t_l - t)\right),$$

$$\chi_k(t) = 1 + r_k \cos\left(\frac{2\pi t}{T}\right).$$

The impact of infected on susceptible individuals.

Finite ICU Capacity

- One of the most fundamental and novel aspects of our model is the fact that **it is capable to account for the potential supply-demand imbalance of the ICU cases**, which can have very severe consequences as far as mortality is concerned
- In fact, one of the main arguments in favor of the lockdown of several major economies is that such a lockdown prevents the potential collapse of the healthcare system
- We shall show below that, given a relatively modest ICU overcapacity, such a collapse is highly unlikely even if only a partial lockdown is implemented instead
- We model the overflow by adjusting coefficients δ

$$\delta_k^{(H)} = \bar{\delta}_k^{(H)} \mathcal{H}\left(C - \sum_{k=1}^{K} F_k\right),$$

$$\delta_k^{(D)} = \bar{\delta}_k^{(D)} + \bar{\delta}_k^{(H)}\left(1 - \mathcal{H}\left(C - \sum_{k=1}^{K} F_k\right)\right)$$

where $\mathcal{H}(.)$ is the Heaviside step function.

Modelling the finite ICU capacity

Example: Main Equations for Two Groups

- Consider the case where $K = 2$, so that there are only two groups
  - the low-risk group (LR, or group 1)
  - the high-risk group (HR, or group 2)
- Since age is an important determinant of which group an individual belongs to, we can assume that relative sizes of the groups are 9:1
- The corresponding equations for group 1 can be written as shown in the right image
- Equations for group 2 follow the same structure

$$
\begin{aligned}
\frac{dS_1}{dt} &= -\Theta_1 a_1 S_1, \\
\frac{dE_1}{dt} &= \Theta_1 a_1 S_1 - \alpha_1 E_1, \\
\frac{dI_1}{dt} &= \alpha_1 E_1 - \left(\beta_1^{(H)} + \beta_1^{(R)} + \beta_1^{(D)}\right) I_1, \\
\frac{dH_1}{dt} &= \beta_1^{(H)} I_1 - \left(\gamma_1^{(F)} + \gamma_1^{(R)} + \gamma_1^{(D)}\right) H_1 + \delta_1^{(H)} F_1, \\
\frac{dF_1}{dt} &= \gamma_1^{(F)} H_1 - \left(\delta_1^{(H)} + \delta_1^{(D)}\right) F_1, \\
\frac{dR_1}{dt} &= \beta_1^{(R)} I_1 + \gamma_1^{(R)} H_1, \\
\frac{dD_1}{dt} &= \beta_1^{(D)} I_1 + \gamma_1^{(D)} H_1 + \delta_1^{(D)} F_1, \\
\frac{dN_1}{dt} &= -\left(\beta_1^{(D)} I_1 + \gamma_1^{(D)} H_1 + \delta_1^{(D)} F_1\right),
\end{aligned}
$$

The main equations for the two-group case

Example: Specific Choices for Two Groups

- Since K=2, we can be more specific regarding coefficients of the main equations
- Namely, we can simplify expressions for θ, and specify splitting of coefficients β, γ, δ
- Mortality rates for the LR and HR groups are very different, 0.18% for group 1, and 2.33% for group 2. Overall, COVID-19's mortality is of order 0.4%, as we shall confirm in the next sections

$$\Theta_1 = \pi_1 \frac{I_1}{N_1} + (1-\pi_1)\frac{(1-\pi_1)a_1 I_1 + (1-\pi_2)a_2 I_2}{(1-\pi_1)a_1 N_1 + (1-\pi_2)a_2 N_2},$$
$$\Theta_2 = \pi_2 \frac{I_2}{N_2} + (1-\pi_2)\frac{(1-\pi_1)a_1 I_1 + (1-\pi_2)a_2 I_2}{(1-\pi_1)a_1 N_1 + (1-\pi_2)a_2 N_2},$$

$$\delta_1^{(H)} = \bar{\delta}_1^{(H)} \mathcal{H}(C-F),$$
$$\delta_1^{(D)} = \bar{\delta}_1^{(D)} + \bar{\delta}_1^{(H)} (1-\mathcal{H}(C-F)),$$
$$\delta_2^{(R)} = \bar{\delta}_2^{(H)} \mathcal{H}(C-F),$$
$$\delta_2^{(D)} = \bar{\delta}_2^{(D)} + \bar{\delta}_2^{(H)} (1-\mathcal{H}(C-F))$$

Disclaimer

- This study is work in progress, based on limited data. The authors will expand and correct results as additional information becomes available
- The views expressed in this document are the authors' and do not necessarily reflect those of the organizations they are affiliated with
- The authors are grateful to Dr. Marsha Lipton and Dr. David Gershon for their invaluable help
- The authors are mathematicians, and their analysis focuses on how epidemics impact society, not individual patients. For individual treatment advice, please consult a licensed physician
-